5.6 Properties of Linear Relations

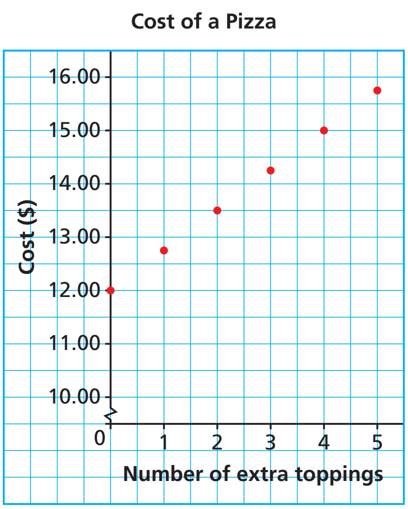
LESSON FOCUS

Identify and represent linear relations in different ways.

Make Connections

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75



What patterns do you see in the table?

Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

How are the patterns in the table shown in the graph?

How can you tell from the table that the graph represents a linear relation?

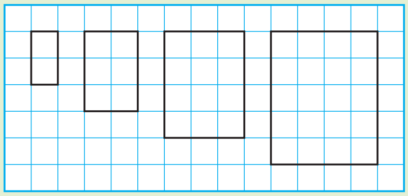
Construct Understanding

TRY THIS

Work with a partner.

You will need 1-cm grid paper.

Use this pattern of rectangles.
This pattern continues.



A. Draw the next two rectangles in the pattern.
Copy and complete each table of values for the 6 rectangles.

Width of Rectangle (cm)	Area (cm^2)
1	
2	

Width of Rectangle (cm)	Perimeter (cm)
1	
2	

B. Which table of values represents a linear relation? How can you tell?

C. Graph the data in each table of values.
Does each graph represent a linear relation?
How do you know?

The cost for a car rental is \$60, plus \$20 for every 100 km driven.
The independent variable is the distance driven and the dependent variable is the cost.
We can identify that this is a linear relation in different ways.

- a table of values

Independent variable → Distance (km); Dependent variable → Cost (\$)

	Distance (km)	Cost (\$)	
	0	60	
+100	100	80	+20
+100	200	100	+20
+100	300	120	+20
+100	400	140	+20

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

Why is it important that the ordered pairs are listed so their first elements are in numerical order?

The graph of a linear relation is a straight line.

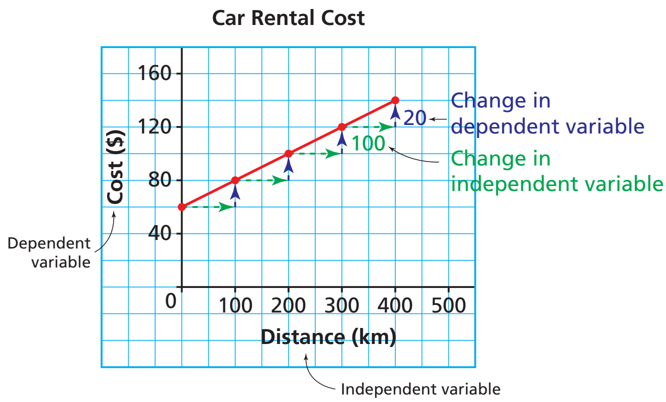
- a set of ordered pairs

$$\{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)\}$$

$+100 \quad +100 \quad +100 \quad +100$

$+20 \quad +20 \quad +20 \quad +20$

- a graph



We can use each representation above to calculate the **rate of change**.
The rate of change can be expressed as a fraction:

$$\frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100\text{ km}}$$

$$= \$0.20/\text{km}$$

The rate of change is \$0.20/km; that is, for each additional 1 km driven, the rental cost increases by 20¢. The rate of change is constant for a linear relation.

We can determine the rate of change from the equation that represents the linear function.
Let the cost be C dollars and the distance driven be d kilometres.
An equation for this linear function is:

$C = 0.20d + 60$

initial amount
independent variable
rate of change
dependent variable

Example 1 Determining whether a Table of Values Represents a Linear Relation

Which table of values represents a linear relation? Justify the answer.

a) The relation between temperature in degrees Celsius, *C*, and temperature in degrees Fahrenheit, *F*

C	*F*
0	32
5	41
10	50
15	59
20	68

b) The relation between the current, *I* amps, and power, *P* watts, in an electrical circuit

I	*P*
0	0
5	75
10	300
15	675
20	1200

SOLUTION

The terms in the first column are in numerical order. So, calculate the change in each variable.

a)

C	Change in *C*	*F*	Change in *F*
0		32	
5	$5 - 0 = 5$	41	$41 - 32 = 9$
10	$10 - 5 = 5$	50	$50 - 41 = 9$
15	$15 - 10 = 5$	59	$59 - 50 = 9$
20	$20 - 15 = 5$	68	$68 - 59 = 9$

Since the changes in both variables are constant, the table of values represents a linear relation.

b)

I	Change in *I*	*P*	Change in *P*
0		0	
5	$5 - 0 = 5$	75	$75 - 0 = 75$
10	$10 - 5 = 5$	300	$300 - 75 = 225$
15	$15 - 10 = 5$	675	$675 - 300 = 375$
20	$20 - 15 = 5$	1200	$1200 - 675 = 525$

The changes in *I* are constant, but the changes in *P* are not constant. So, the table of values does not represent a linear relation.

CHECK YOUR UNDERSTANDING

1. Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, *n*, and time, *t* minutes.

t	*n*
0	1
20	2
40	4
60	8
80	16
100	32

b) The relation between the amount of goods and services tax charged, *T* dollars, and the amount of the purchase, *A* dollars

A	*T*
60	3
120	6
180	9
240	12
300	15

[Answers: a) not linear b) linear]

What other strategies could you use to check whether each table of values represents a linear relation?

When an equation is written using the variables x and y, x represents the independent variable and y represents the dependent variable.

Example 2 Determining whether an Equation Represents a Linear Relation

a) Graph each equation.

i) $y = -3x + 25$ **ii)** $y = 2x^2 + 5$

iii) $y = 5$ **iv)** $x = 1$

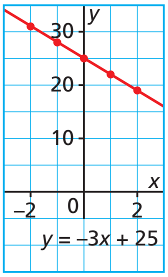
b) Which equations in part a represent linear relations? How do you know?

SOLUTION

a) Create a table of values, then graph the relation.

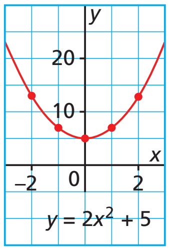
i) $y = -3x + 25$

x	y
−2	31
−1	28
0	25
1	22
2	19



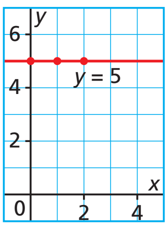
ii) $y = 2x^2 + 5$

x	y
−2	13
−1	7
0	5
1	7
2	13



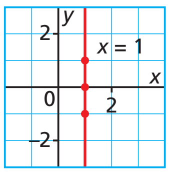
iii) $y = 5$

x	y
0	5
1	5
2	5



iv) $x = 1$

x	y
1	−1
1	0
1	1



b) The graphs in parts i, iii, and iv are straight lines, so their equations represent linear relations; that is, $y = -3x + 25$, $y = 5$, and $x = 1$.
The graph in part ii is not a straight line, so its equation does not represent a linear relation.

CHECK YOUR UNDERSTANDING

2. a) Graph each equation.

i) $x = -2$

ii) $y = x + 25$

iii) $y = 25$

iv) $y = x^2 + 25$

b) Which equations in part a represent linear relations? How do you know?

[Answers: b) $x = -2$; $y = x + 25$; $y = 25$]

Example 3 Identifying a Linear Relation

Which relation is linear? Justify the answer.

a) A new car is purchased for $24 000. Every year, the value of the car decreases by 15%. The value is related to time.

b) For a service call, an electrician charges a $75 flat rate, plus $50 for each hour he works. The total cost for service is related to time.

SOLUTION

Create a table of values, then check to see if the relation is linear.

a) Every year, the value decreases by 15%.
The value of the car is:
100% − 15% = 85% of its previous value
So, multiply each value by 0.85.

	Time (years)	Value ($)	
	0	24 000	
+1	1	20 400	−3600
+1	2	17 340	−3060
+1	3	14 739	−2601

There is a constant change of 1 in the 1st column, but the differences in the 2nd column are not constant. So, the relation is not linear.

b) After the first hour, the cost increases by $50 per hour.

	Time (h)	Cost ($)	
	0	75	
+1	1	125	+50
+1	2	175	+50
+1	3	225	+50
+1	4	275	+50

There is a constant change of 1 in the 1st column and a constant change of 50 in the 2nd column, so the relation is linear.

CHECK YOUR UNDERSTANDING

3. Which relation is linear? Justify your answer.

a) A dogsled moves at an average speed of 10 km/h along a frozen river. The distance travelled is related to time.

b) The area of a square is related to the side length of the square.

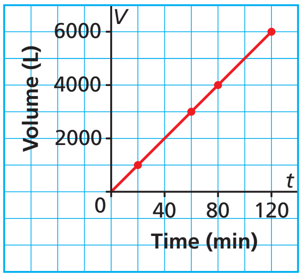
[Answers: a) linear
b) not linear]

What equation could you write for the linear relation in part b?

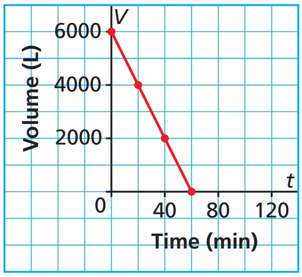
Example 4 Determining the Rate of Change of a Linear Relation from Its Graph

A water tank on a farm near Swift Current, Saskatchewan, holds 6000 L.
Graph A represents the tank being filled at a constant rate.
Graph B represents the tank being emptied at a constant rate.

Graph A
Filling a Water Tank



Graph B
Emptying a Water Tank



a) Identify the independent and dependent variables.

b) Determine the rate of change of each relation, then describe what it represents.

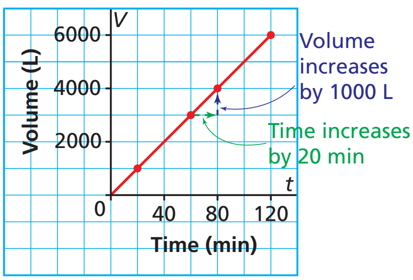
SOLUTION

For Graph A

a) The independent variable is the time, *t*.
The dependent variable is the volume, *V*.

b) Choose two points on the line. Calculate the change in each variable from one point to the other.

Graph A
Filling a Water Tank



Change in volume:
4000 L − 3000 L = 1000 L

Change in time:
80 min − 60 min = 20 min

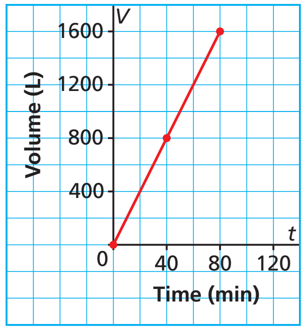
Rate of change: $\frac{1000\text{ L}}{20\text{ min}} = 50\text{ L/min}$

The rate of change is positive so the volume is increasing with time.
Every minute, 50 L of water are added to the tank.

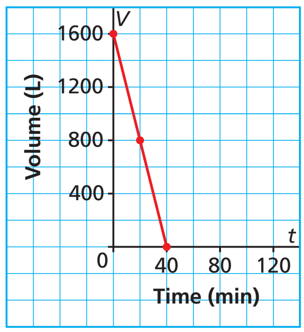
CHECK YOUR UNDERSTANDING

4. A hot tub contains 1600 L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.

Graph A
Filling a Hot Tub



Graph B
Emptying a Hot Tub



a) Identify the dependent and independent variables.

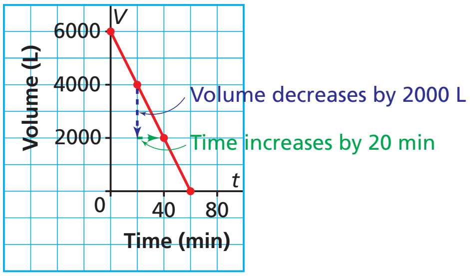
b) Determine the rate of change of each relation, then describe what it represents.

[Answers: Graph A a) *V*, *t* b) 20 L/min
Graph B a) *V*, *t* b) −40 L/min]

For Graph B

a) The independent variable is the time, *t*.
The dependent variable is the volume, *V*.

b) Choose two points on the line.
Calculate the change in each variable from one point to the other.



Change in volume: 2000 L − 4000 L = −2000 L
Change in time: 40 min − 20 min = 20 min

Rate of change: $\frac{-2000 \text{ L}}{20 \text{ min}} = -100 \text{ L/min}$

The rate of change is negative so the volume is decreasing with time.
Every minute, 100 L of water are removed from the tank.

Discuss the Ideas

1. How can you tell from each format whether a relation is linear?
- a description in words
- a set of ordered pairs
- a table of values
- an equation
- a graph

2. What is "rate of change"? How can you use each format in question 1 to determine the rate of change of a linear relation?

Exercises

A

3. Which tables of values represent linear relations? Explain your answers.

a)

Time (min)	Distance (m)
0	10
2	50
4	90
6	130

b)

Time (s)	Speed (m/s)
0	10
1	20
2	40
3	80

c)

Speed (m/s)	Time (s)
15	7.5
10	5
5	2.5
0	0

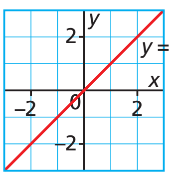
d)

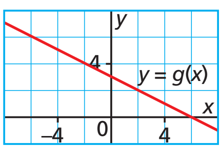
Distance (m)	Speed (m/s)
4	2
16	4
1	1
9	3

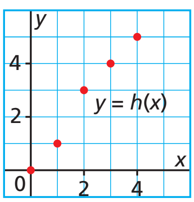
4. Which sets of ordered pairs represent linear relations? Explain your answers.

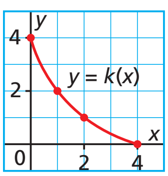
a) $\{(3, 11), (5, 9), (7, 7), (9, 5)\}$
b) $\{(-2, 3), (0, 1), (2, -3), (4, -7)\}$
c) $\{(1, 1), (1, 3), (2, 1), (2, 3)\}$

5. Which graphs represent linear relations? How do you know?

a) 

b) 

c) 

d) 

B

6. a) Create a table of values when necessary, then graph each relation.

i) $y = 2x + 8$ **ii)** $y = 0.5x + 12$
iii) $y = x^2 + 8$ **iv)** $y = 2x$
v) $x = 7$ **vi)** $x + y = 6$

b) Which equations in part a represent linear relations? How do you know?

7. For each relation below:

i) Identify the dependent and independent variables.
ii) Use the table of values to determine whether the relation is linear.
iii) If the relation is linear, determine its rate of change.

a) The distance required for a car to come to a complete stop after its brakes are applied is the *braking distance*. The braking distance, d metres, is related to the speed of the car, s kilometres per hour, when the brakes are first applied.

s (km/h)	d (m)
50	13
60	20
70	27
80	35

b) The altitude of a plane, a metres, is related to the time, t minutes, that has elapsed since it started its descent.

t (min)	a (m)
0	12 000
2	11 600
4	11 200
6	10 800
8	10 400

8. In a hot-air balloon, a chart shows how the distance to the horizon, d kilometres, is related to the height of the balloon, h metres.

h (m)	d (km)
5	8
10	11
30	20
50	25
100	36

a) Graph these data.
b) Is the relation linear? What strategy did you use?

9. Earth rotates through approximately 360° every 24 h. The set of ordered pairs below describes the rotation. The first coordinate is the time in hours, and the second coordinate is the approximate angle of rotation in degrees. Describe two strategies you could use to determine if this relation is linear.
$\{(0, 0), (6, 90), (12, 180), (18, 270), (24, 360)\}$

10. Sophie and 4 of her friends plan a trip to the Edmonton Chante for one night. The hotel room is \$95 for the first 2 people, plus \$10 for each additional person in the room. The total cost is related to the number of people. Is the relation linear? How do you know?

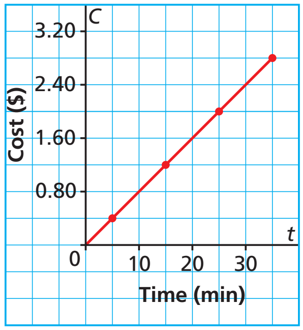
11. A skydiver jumps from an altitude of 3600 m. For the first 12 s, her height in metres above the ground is described by this set of ordered pairs:
$\{(0, 3600), (4, 3526), (8, 3353.5), (12, 3147.5)\}$
For the next 21 s, her height above the ground is described by this set of ordered pairs:
$\{(15, 2988.5), (21, 2670.5), (27, 2352.5), (33, 2034.5)\}$
Determine whether either set of ordered pairs represents a linear relation. Explain.

12. The cost, C dollars, to rent a hall for a banquet is given by the equation $C = 550 + 15n$, where n represents the number of people attending the banquet.
a) Explain why the equation represents a linear relation.
b) State the rate of change. What does it represent?

13. A safety flare is shot upward from the top of a cliff 200 m above sea level. An equation for the height of the flare, d metres, above sea level t seconds after the flare is fired, is given by the equation $d = -4.9t^2 + 153.2t + 200$. Describe two strategies you could use to determine whether this relation is linear.

14. This graph represents Jerome's long distance phone call to his pen pal in Nunavut. Jerome is charged a constant rate.

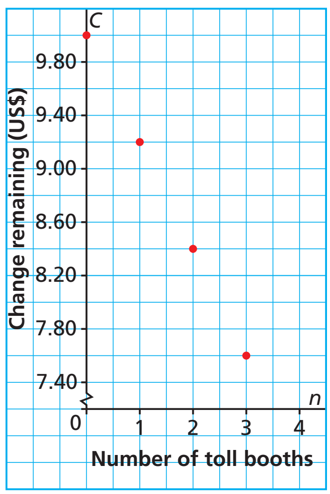
The Cost of Jerome's Phone Call



a) Identify the dependent and independent variables.
b) Determine the rate of change, then describe what it represents.

15. Kashala takes a cross-country trip from her home in Lethbridge through the United States. In Illinois, she drives on a toll highway. This graph represents the cost of Kashala's drive on the toll highway. She is charged a constant amount at each toll booth and she starts with US\$10 in change. Determine the rate of change, then describe what it represents.

Kashala's Drive on the Toll Highway



16. Match each description of a linear relation with its equation and set of ordered pairs below. Explain your choices.

a) The amount a person earns is related to her hourly wage.

b) The cost of a banquet is related to a flat fee plus an amount for each person who attends.

c) The volume of gas in a car's gas tank is related to the distance driven since the time when the tank was filled.

Equation 1: $y = 500 + 40x$
Equation 2: $y = 35 - 0.06x$
Equation 3: $y = 20x$
Set A: $\{(100, 29), (200, 23), (300, 17), (400, 11)\}$
Set B: $\{(1, 20), (5, 100), (10, 200), (15, 300)\}$
Set C: $\{(0, 500), (40, 2100), (80, 3700), (100, 4500)\}$

17. a) Which situations represent linear relations? Explain how you know.

i) A hang glider starts her descent at an altitude of 2000 m. She descends at a constant speed to an altitude of 1500 m in 10 min.

ii) A population of bacteria triples every hour for 4 h.

iii) A taxi service charges a \$5 flat fee plus \$2 for each kilometre travelled.

iv) The cost to print each yearbook is \$5. There is a start up fee of \$500 to set up the printing press.

v) An investment increases in value by 12% each year.

b) For each linear relation in part a, identify:

- the dependent and independent variables
- the rate of change and explain what it represents

C

18. Identify the measurement formulas that represent linear relations. Explain how you know.

a) Perimeter, P, of an equilateral triangle with side length s: $P = 3s$

b) Surface area, A, of a cube with edge length s: $A = 6s^2$

c) Volume, V, of a sphere with radius r: $V = \frac{4}{3}\pi r^3$

d) Circumference, C, of a circle with diameter d: $C = \pi d$

e) Area, A, of a circle with radius r: $A = \pi r^2$

19. Here are two equations that can be used to model the value, V dollars, of a \$24 000 truck as it depreciates over n years:
$V = 24\,000 - 2000n$ and $V = 24\,000(0.2^n)$

a) Which equation represents a linear relation? Justify your answer.

b) For the linear relation, state the rate of change. What does it represent?

20. You can estimate the distance in kilometres between you and a distant storm by measuring the time in seconds between seeing a lightning flash and hearing the thunder, then dividing by 3. This works because sound travels at approximately 0.3 km/s. Is this relation between distance and time linear? Justify your answer.

21. A berry patch is to be harvested. Is the relation between the time it will take to harvest the patch and the number of pickers needed linear? Justify your answer.

22. Which statements are true? Use examples to justify your answers.

a) A relation described by exactly two ordered pairs is always linear.

b) An equation of the form $Ax + By = C$ for non-zero constants, A, B, and C, always represents a linear function.

c) An equation of the form $y = Cx^2$ for a non-zero constant C, always represents a linear function.

d) An equation of the form $x = C$ for a constant C, always represents a linear relation.

e) A linear relation is always a linear function.

Reflect

List three different strategies you can use to tell whether a relation is linear. Include an example with each strategy.

5.7 Interpreting Graphs of Linear Functions

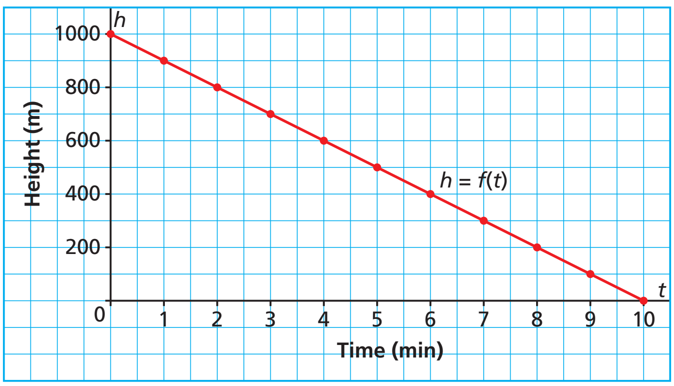
LESSON FOCUS

Use intercepts, rate of change, domain, and range to describe the graph of a linear function.

Make Connections

Float planes fly into remote lakes in Canada's Northern wilderness areas for ecotourism. This graph shows the height of a float plane above a lake as the plane descends to land.

Height of a Float Plane



Where does the graph intersect the vertical axis? What does this point represent?

Where does the graph intersect the horizontal axis?
What does this point represent?

What is the rate of change for this graph? What does it represent?

Construct Understanding

TRY THIS

Work in a group.

You will need grid paper.

Dogsled tours are run between Armstrong cabin and Irving cabin. The cabins are 100 km apart.

Dogsled team 1 travels at an average speed of 20 km/h and starts its tour at Armstrong cabin.

Dogsled team 2 travels at an average speed of 25 km/h and starts its tour at Irving cabin.

One pair of students chooses team 1 and the other pair chooses team 2.

A. Copy and complete the table to show the distance from Irving cabin at different times on the tour.

Team 1

Time (h)	Distance from Irving Cabin (km)
0	100
1	

Team 2

Time (h)	Distance from Irving Cabin (km)
0	0
1	

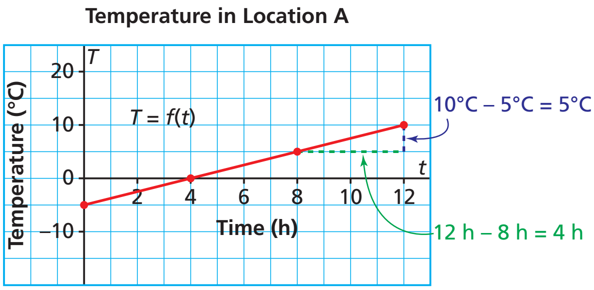
B. Draw a graph to show the distance from Irving cabin as a function of time.

C. Share your results with the other pair of students.

- How are the graphs the same? How are they different?
- Identify where each graph intersects the vertical and horizontal axes. What do these points represent?
- Determine the rate of change for each graph. What does it represent?
- What are the domain and range for each graph?

Any graph of a line that is not vertical represents a function.
We call these functions **linear functions**.

Each graph below shows the temperature, T degrees Celsius, as a function of time, t hours, for two locations.



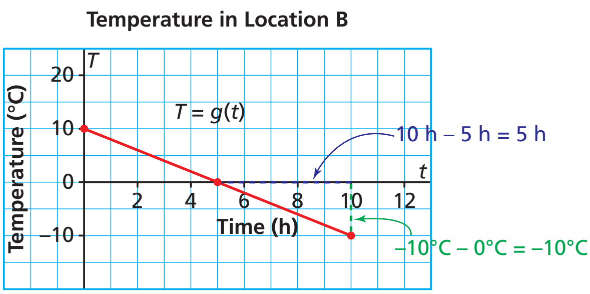
The point where the graph intersects the horizontal axis has coordinates (4, 0). The **horizontal intercept** is 4. This point of intersection represents the time, after 4 h, when the temperature is 0°C.

The point where the graph intersects the vertical axis has coordinates (0, –5). The **vertical intercept** is –5. This point of intersection represents the initial temperature, –5°C.

The *domain* is: $0 \leq t \leq 12$
The *range* is: $-5 \leq T \leq 10$
The *rate of change* is: $\dfrac{\text{change in } T}{\text{change in } t} = \dfrac{5°\text{C}}{4\,\text{h}}$

$= 1.25°\text{C/h}$

The rate of change is positive because the temperature is increasing over time.



The point where the graph intersects the horizontal axis has coordinates (5, 0). The *horizontal intercept* is 5. This point of intersection represents the time, after 5 h, when the temperature is 0°C.

The point where the graph intersects the vertical axis has coordinates (0, 10). The *vertical intercept* is 10. This point of intersection represents the initial temperature, 10°C.

The *domain* is: $0 \leq t \leq 10$
The *range* is: $-10 \leq T \leq 10$

The *rate of change* is: $\dfrac{\text{change in } T}{\text{change in } t} = \dfrac{-10°\text{C}}{5\,\text{h}}$

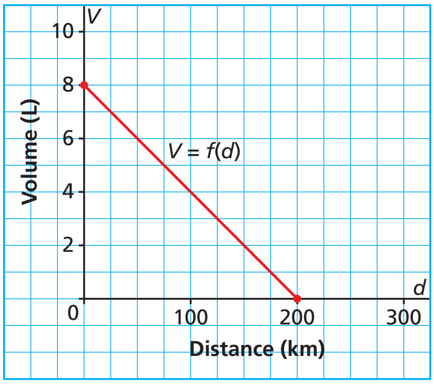
$= -2°\text{C/h}$

The rate of change is negative because the temperature is decreasing over time.

Example 1 Determining Intercepts, Domain, and Range of the Graph of a Linear Function

This graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.

Volume of Gas in a Scooter



a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

b) What are the domain and range of this function?

SOLUTION

a) On the vertical axis, the point of intersection has coordinates (0, 8). The vertical intercept is 8. This point of intersection represents the volume of gas in the tank when the distance travelled is 0 km; that is, the capacity of the gas tank: 8 L

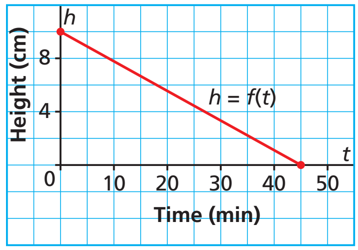
On the horizontal axis, the point of intersection has coordinates (200, 0). The horizontal intercept is 200. This point of intersection is the distance travelled until the volume of gas is 0 L; that is, the distance the scooter can travel on a full tank of gas: 200 km

b) The domain is the set of possible values of the distance travelled:
$0 \leq d \leq 200$
The range is the set of possible values of the volume of fuel:
$0 \leq V \leq 8$

CHECK YOUR UNDERSTANDING

1. This graph shows how the height of a burning candle changes with time.

Height of a Burning Candle



a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

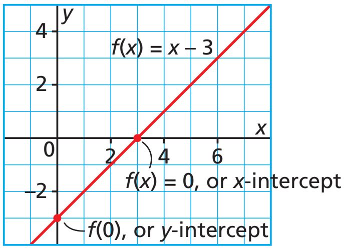
b) What are the domain and range of this function?

[Answers: a) (0, 10), 10; (45, 0), 45
b) domain: $0 \leq t \leq 45$; range: $0 \leq h \leq 10$]

Are there any restrictions on the domain and range? Explain.

What is the fuel consumption in litres per 100 km?

We can use the intercepts to graph a linear function written in function notation.
To determine the y-intercept, evaluate $f(x)$ when $x = 0$; that is, evaluate $f(0)$.
To determine the x-intercept, determine the value of x when $f(x) = 0$.



The x-coordinate of the point where a graph intersects the x-axis is called the ***x*-intercept**, or the **horizontal intercept**.

The y-coordinate of the point where a graph intersects the y-axis is called the ***y*-intercept**, or the **vertical intercept**.

Example 2 Sketching a Graph of a Linear Function in Function Notation

Sketch a graph of the linear function $f(x) = -2x + 7$.

SOLUTION

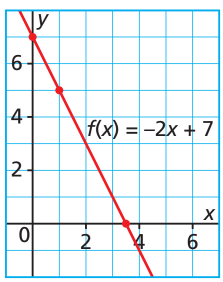
$f(x) = -2x + 7$
Since the function is linear, its graph is a straight line.

Determine the y-intercept:
When $x = 0$,
$f(0) = -2(0) + 7$
$f(0) = 7$

Determine the x-intercept:
When $f(x) = 0$,

$$0 = -2x + 7$$
$$-7 = -2x + 7 - 7$$
$$-7 = -2x$$
$$x = \frac{-7}{-2}$$
$$x = \frac{7}{2}$$

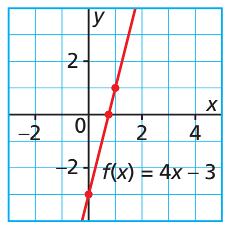
Determine the coordinates of a third point on the graph.
When $x = 1$,
$f(1) = -2(1) + 7$
$f(1) = 5$
Plot the points $(0, 7)$, $\left(\frac{7}{2}, 0\right)$, and $(1, 5)$, then draw a line through them.



CHECK YOUR UNDERSTANDING

2. Sketch a graph of the linear function $f(x) = 4x - 3$.

Answer:

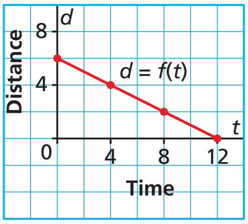


What other strategy could you use to graph the function? Which strategy would be more efficient?

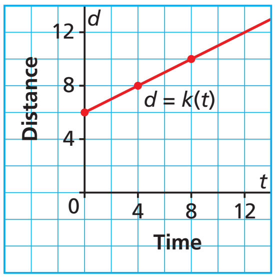
Example 3 Matching a Graph to a Given Rate of Change and Vertical Intercept

Which graph has a rate of change of $\frac{1}{2}$ and a vertical intercept of 6? Justify the answer.

a)

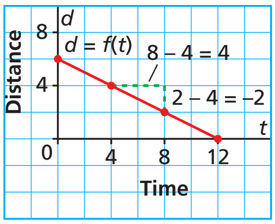


b)



SOLUTION

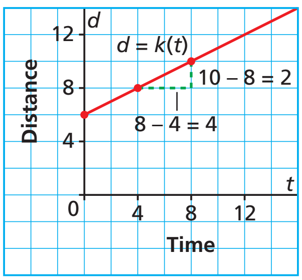
a) The graph of $d = f(t)$ has a vertical intercept of 6.



The rate of change is: $\frac{-2}{4} = -\frac{1}{2}$

So, it is not the correct graph.

b) The graph of $d = k(t)$ has a vertical intercept of 6.



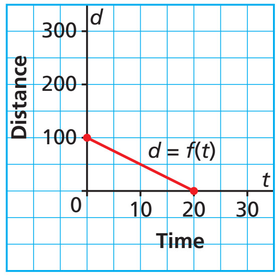
The rate of change is: $\frac{2}{4} = \frac{1}{2}$

So, this is the correct graph.

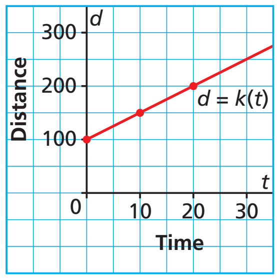
CHECK YOUR UNDERSTANDING

3. Which graph has a rate of change of -5 and a vertical intercept of 100? Justify your answer.

a)



b)

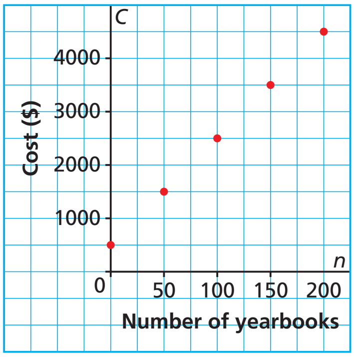


[Answer: the graph in part a]

Example 4 Solving a Problem Involving a Linear Function

This graph shows the cost of publishing a school yearbook for Collège Louis-Riel in Winnipeg.

Cost of Publishing a Yearbook



The budget for publishing costs is \$4200. What is the maximum number of books that can be printed?

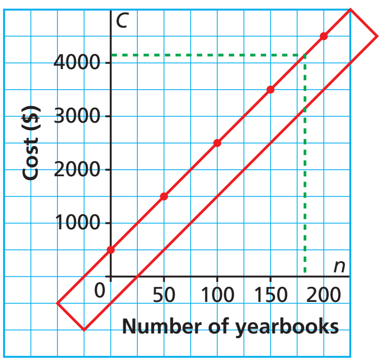
SOLUTIONS

Method 1

To estimate the number of yearbooks that can be printed for \$4200, use the graph.

From 4200 on the *C*-axis, draw a horizontal line to the graph, then a vertical line to the *n*-axis.

Cost of Publishing a Yearbook



Use a straightedge to help.

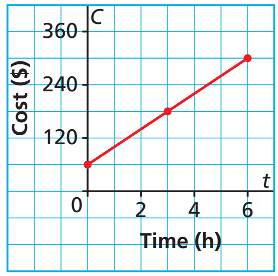
From the graph, about 180 yearbooks can be printed.

(Solution continues.)

CHECK YOUR UNDERSTANDING

4. This graph shows the total cost for a house call by an electrician for up to 6 h work.

Cost of an Electrician's House Call



The electrician charges \$190 to complete a job. For how many hours did she work?

[Answer: $3\frac{1}{4}$ h]

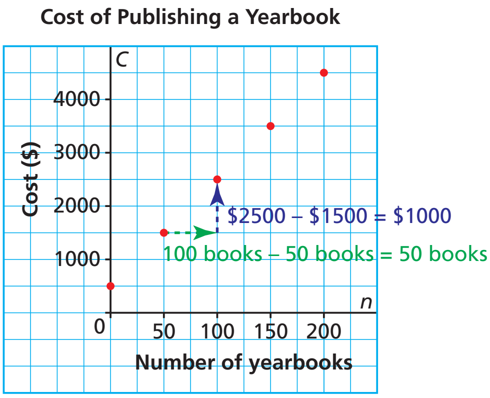
Why are the points on this graph not joined?

What are the domain and range of this function?

Method 2

The set-up cost is the cost when the number of books printed is 0.
This is the vertical intercept of the graph, which is 500.
The set-up cost is $500.

The increase in cost for each additional book printed is the rate of change of the function. Determine the change in each variable.



The graph shows that for every 50 books printed, the cost increases by $1000.

The rate of change is: $\dfrac{\$1000}{50\text{ books}} = \$20/\text{book}$

The increase in cost for each additional book published is $20.

An equation that represents this situation is: $C = 20n + 500$

To determine the maximum number of yearbooks that can be printed, use the equation:

$$\begin{aligned} C &= 20n + 500 && \text{Substitute: } C = 4200 \\ 4200 &= 20n + 500 && \text{Solve for } n. \\ 4200 - 500 &= 20n + 500 - 500 \\ 3700 &= 20n \\ \frac{3700}{20} &= \frac{20n}{20} \\ 185 &= n \end{aligned}$$

The maximum number of yearbooks that can be printed is 185.

What is an advantage of using each method?

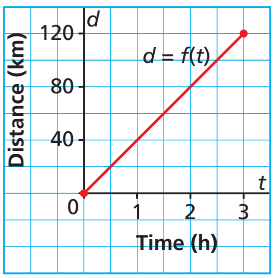
Discuss the Ideas

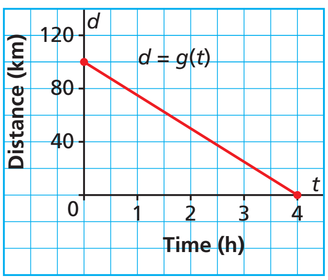
1. What information do the vertical and horizontal intercepts provide about a linear function? Use an example to explain.
2. How can you tell from a graph whether a linear function has a positive or negative rate of change?
3. When a situation can be described by a linear function, why doesn't it matter which pair of points you choose to determine the rate of change?

Exercises

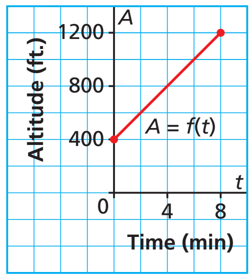
A

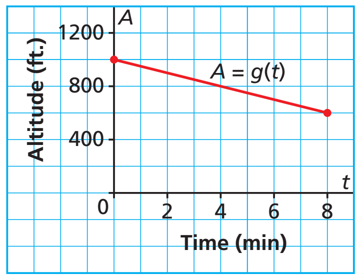
4. Each graph below shows distance, d kilometres, as a function of time, t hours. For each graph:
   i) Determine the vertical and horizontal intercepts. Write the coordinates of the points where the graph intersects the axes.
   ii) Determine the rate of change.
   iii) Determine the domain and range.

a) 

b) 

5. Each graph shows the altitude, A feet, of a small plane as a function of time, t minutes.
   For each graph:
   i) Determine the vertical intercept. Write the coordinates of the point where the graph intersects the axis.
   ii) Determine the rate of change.
   iii) Determine the domain and range.

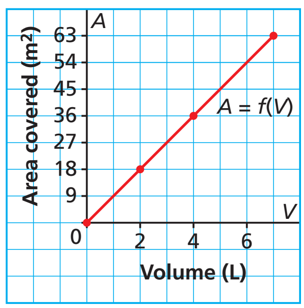
a) 

b) 

B

6. Sketch a graph of each linear function.
   a) $f(x) = 4x + 3$
   b) $g(x) = -3x + 5$
   c) $h(x) = 9x - 2$
   d) $k(x) = -5x - 2$

7. This graph shows the area, A square metres, that paint covers as a function of its volume, V litres.



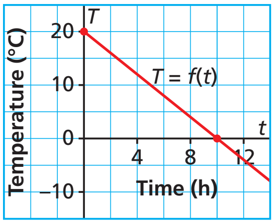
   a) What is the rate of change? What does it represent?
   b) What area is covered by 6 L of paint?
   c) What volume of paint would cover 45 m^2?

8. The graphs below show the temperature, T degrees Celsius, as a function of time, t hours, at different locations.

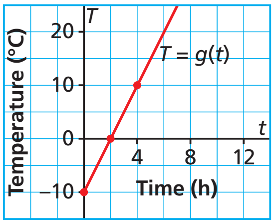
a) Which graph has a rate of change of 5°C/h and a vertical intercept of −10°C?

b) Which graph has a rate of change of −10°C/h and a vertical intercept of 20°C?

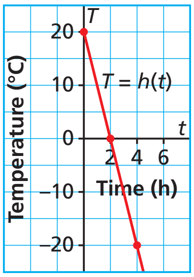
i)



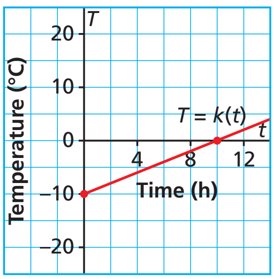
ii)



iii)

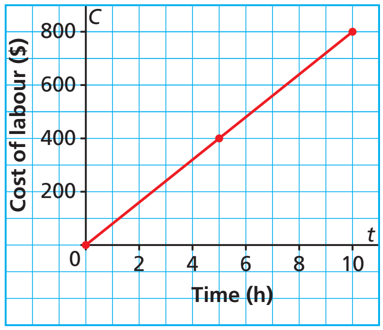


iv)



9. St. Adolphe, Manitoba, is located in the flood plain of the Red River. To help prevent flooding, backhoes were used to build dikes around houses and farms in the town. This graph shows the labour costs for running a backhoe.

Cost of Running a Backhoe



a) Determine the vertical and horizontal intercepts. Write the coordinates of the point where the graph intersects the axes. Describe what the point represents.

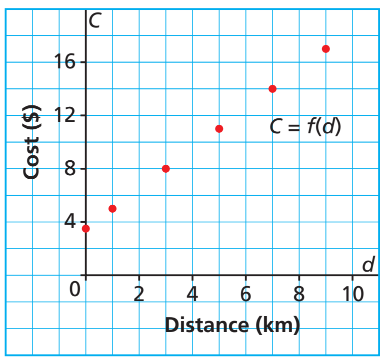
b) Determine the rate of change. What does it represent?

c) Write the domain and range.

d) What is the cost to run the backhoe for 7 h?

e) For how many hours is the backhoe run when the cost is $360?

10. This graph shows the cost for a cab at Eagle Taxi Cabs. The cost, C dollars, is a function of the distance travelled, d kilometres.



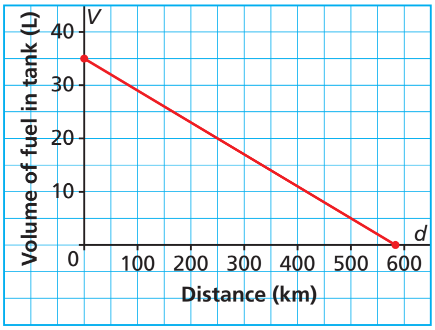
a) Determine the rate of change. What does it represent?

b) What is the cost when the distance is 7 km?

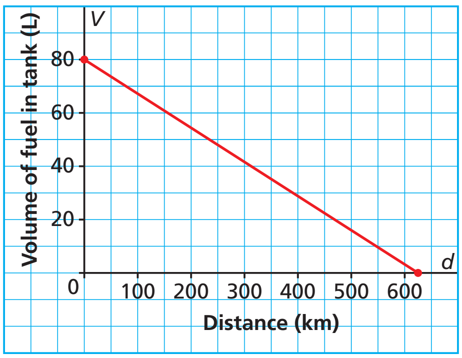
c) What is the distance when the cost is $9.50?

11. A Smart car and an SUV have full fuel tanks, and both cars are driven on city roads until their tanks are nearly empty. The graphs show the fuel consumption for each vehicle.

Fuel Consumption of a Smart Car



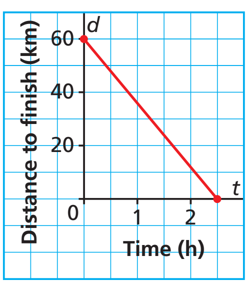
Fuel Consumption of an SUV



Use the graphs to explain why the Smart car is more economical to drive than the SUV.

12. This graph shows the distance to the finish line, d kilometres, as a function of time, t hours, for one dogsled in a race near Churchill, Manitoba.

Dogsled Race



a) What was the length of time it took the dogsled to finish the race?

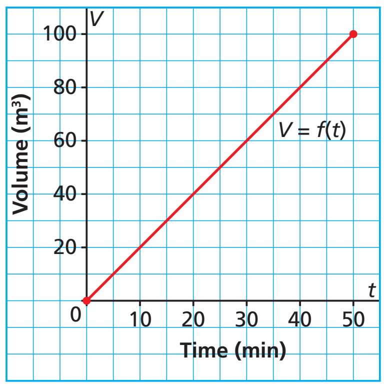
b) What was the average speed of the dogsled?

c) How long was the race in kilometres?

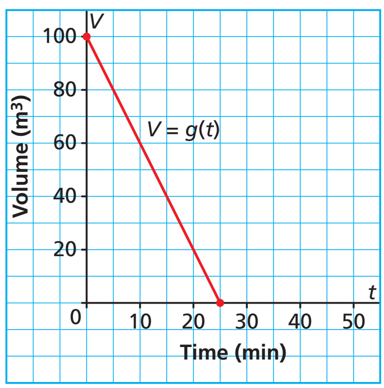
d) What time did it take for the dogsled to complete $\frac{2}{3}$ of the race?

13. The capacity of each of 2 fuel storage tanks is 100 m^3. Graph A represents the volume of fuel in one tank as a function of time as the tank is filled. Graph B represents the volume of fuel in another tank as a function of time as the tank is emptied.

Graph A



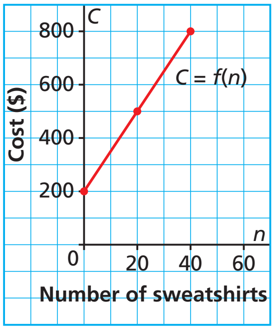
Graph B



a) Does it take longer to fill the empty tank or empty the full tank? How do you know?

b) In the time it takes for one tank to be half empty, about how much fuel would be in a tank that was being filled from empty?

14. Ballenas School places an order for school sweatshirts with its logo of a killer whale on the back. This graph shows the cost of the sweatshirts, C dollars, as a function of the number ordered, n.



a) The number of sweatshirts cannot be a fraction or decimal. Why do you think the points on the graph are joined?

b) i) About how many sweatshirts can be bought for $700?

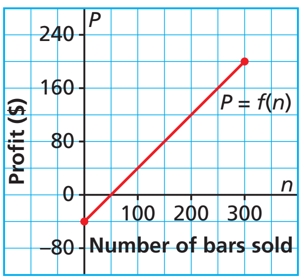
ii) Suppose one more sweatshirt was ordered. What would be the increase in cost?

15. Sketch a graph of each linear function for positive values of the independent variable.

a) $f(x) = 5 - 2.5x$ b) $g(t) = 85t$

c) $h(n) = 750 + 55n$ d) $V(d) = 55 - 0.08d$

16. Northlands School Outdoor Club had a fund-raiser to help purchase snowshoes. The club had 300 power bars to sell. This graph shows the profit made from selling power bars.

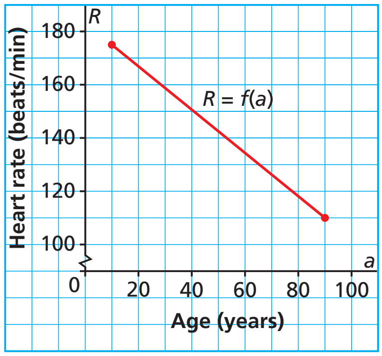


a) What is the profit on each bar sold? How do you know?

b) Determine the intercepts. What does each represent?

c) Describe the domain and range for the function. Why would you not want to list all the values in the range?

17. This graph shows the recommended maximum heart rate of a person, R beats per minute, as a function of her or his age, a years, for a stress test.



a) Why are there no intercepts on this graph?

b) What is the rate of change? What does it represent?

c) At what age is the recommended maximum heart rate 120 beats/min?

d) What is the approximate recommended maximum heart rate for a person aged 70?

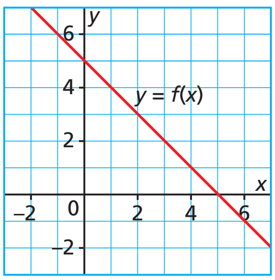
C

18. Two graphs that relate two real numbers x and y in different ways are shown below.
For each graph:

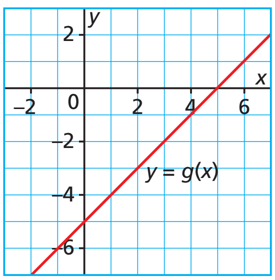
i) State the x- and y-intercepts.

ii) Use the intercepts to describe how x and y are related.

a)



b)



19. a) Sketch a graph of the linear function $d = f(t)$ that satisfies these conditions: $f(1.5) = 127.5$ and $f(3.5) = 297.5$

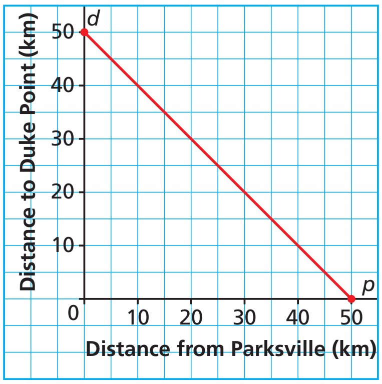
b) Determine $f(5)$.

c) Determine t when $f(t) = 212.5$.

d) Suggest a context for this linear function.

20. The distance between Parksville and the Duke Point Ferry Terminal on Vancouver Island is 50 km. A person drives from Parksville to the ferry terminal.

Distance from Parksville and to Duke Point



a) What do the intercepts represent? Why are they equal?

b) What is the rate of change? Why does it not have units? What does it indicate?

c) How would interchanging the dependent and independent variables change the graph?

d) Suppose the distance between two towns A and B is k kilometres. Describe the graph of the function, "Distance to A as a function of distance from B". State the intercepts, domain and range, and the rate of change.

Reflect

Explain why knowing the intercepts and the rate of change of the graph of a linear function may be helpful when you solve problems. Include examples in your explanation.

THE WORLD OF MATH

Historical Moment: Theano

Theano was one of the first known woman mathematicians. Her husband was Pythagoras, perhaps the most famous mathematician of all time. Theano lived in the 6th century B.C.E. in what is now southern Italy. She wrote many articles on mathematics, as well as on physics, medicine, astronomy, and child psychology. Her most famous work was on the development of the golden ratio and the golden rectangle.

STUDY GUIDE

CONCEPT SUMMARY

Big Ideas	Applying the Big Ideas
■ A relation associates the elements of one set with the elements of another set.	This means that: ■ A relation may be represented as: a rule, a table, a set of ordered pairs, an arrow diagram, and a graph. The set of first elements is the domain and the set of related second elements is the range.
■ A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.	■ For a function, each element of the domain is associated with exactly one element of the range.
■ A linear function has a constant rate of change and its graph is a non-vertical straight line.	■ For a linear function, a constant change in the independent variable results in a constant change in the dependent variable, and any vertical line drawn through the graph intersects the graph at no more than one point.

Reflect on the Chapter

- What is a relation? What is a function? Create a graphic organizer to show their common characteristics, and those that are unique.
- How can the properties of linear functions be used to solve real-world problems? Include examples with your explanation.

SKILLS SUMMARY

<table>
<tr><th>Skill</th><th>Description</th><th>Example</th></tr>
<tr>
<td>Determine the domain and range of a function.

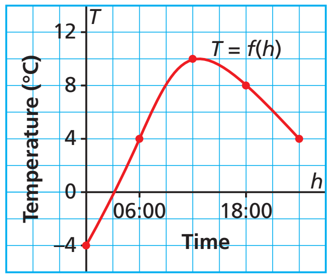
[5.2, 5.4, 5.5, 5.7]</td>
<td>The domain is the set of first elements of the ordered pairs. The range is the set of second elements.

For a graph, the domain is the set of values of the independent variable. The range is the set of values of the dependent variable.</td>
<td>

$\{(-1, 3), (0, 5), (1, 7), (2, 9), (3, 11)\}$

For this set of ordered pairs,
the domain is: $\{-1, 0, 1, 2, 3\}$;
the range is: $\{3, 5, 7, 9, 11\}$

For the graph below:
The domain is all possible times in one day.
The range is: $-4 \leq T \leq 10$



</td>
</tr>
<tr>
<td>Determine the rate of change of the graph of a linear function.

[5.6, 5.7]</td>
<td>

The rate of change is:

$\dfrac{\text{change in dependent variable}}{\text{change in independent variable}}$

The rate of change is positive when the graph goes up to the right. The rate of change is negative when the graph goes down to the right.

</td>
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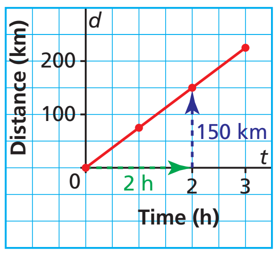
Distance against Time



The rate of change is:

$\dfrac{150\text{ km}}{2\text{ h}} = 75\text{ km/h}$

</td>
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<tr>
<td>Determine the intercepts of the graph of a linear function.

[5.7]</td>
<td>

The x-intercept is the value of x when y or $f(x)$ is 0.

The y-intercept is the value of y when x is 0.

</td>
<td>

For the linear function
$f(x) = -2x + 5$,
When $f(x) = 0$:
$0 = -2x + 5$
$2x = 5$
$x = 2.5$
The x-intercept is 2.5.
When $x = 0$:
$f(0) = -2(0) + 5$
$f(0) = 5$
The y-intercept is 5.

</td>
</tr>
</table>

REVIEW

5.1

1. This table shows some Northwest Coast artists and their cultural heritage.

Artist	Heritage
Bob Dempsey	Tlingit
Dorothy Grant	Haida
Bill Helin	Tsimshian
John Joseph	Squamish
Judith P. Morgan	Gitxsan
Bill Reid	Haida
Susan Point	Salish

a) Describe the relation in words.
b) Represent this relation:
  i) as a set of ordered pairs
  ii) as an arrow diagram

2. Here is a list of some chemical elements and their atomic numbers:
hydrogen (1), oxygen (8), iron (26),
chlorine (17), carbon (6), silver (47)
For each association below, use these data to represent a relation in different ways.
a) has an atomic number of
b) is the atomic number of

5.2

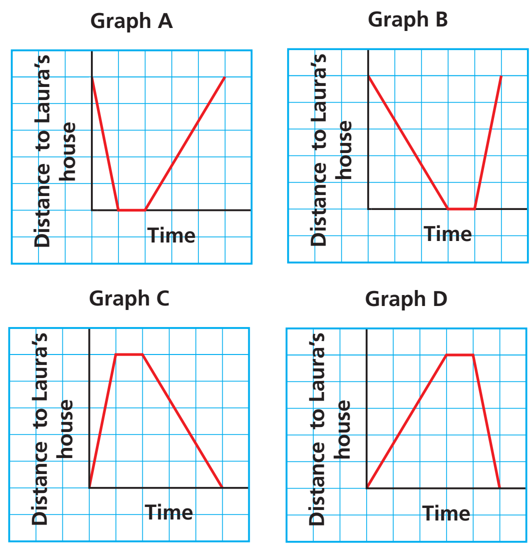
3. Which sets of ordered pairs represent functions? What strategies did you use to find out?
a) $\{(4, 3), (4, 2), (4, 1), (4, 0)\}$
b) $\{(2, 4), (-2, 4), (3, 9), (-3, 9)\}$
c) $\{(2, 8), (3, 12), (4, 16), (5, 20)\}$
d) $\{(5, 5), (5, -5), (-5, 5), (-5, -5)\}$

4. Write in function notation.
a) $y = -4x + 9$ **b)** $C = 12n + 75$
c) $D = -20t + 150$ **d)** $P = 4s$

5. The function $P(n) = 5n - 300$ describes the profit, P dollars, for a school dance when n students attend.
a) Write the function as an equation in 2 variables.
b) Identify the independent variable and the dependent variable. Justify your choices.
c) Determine the value of $P(150)$. What does this number represent?
d) Determine the value of n when $P(n) = 700$. What does this number represent?

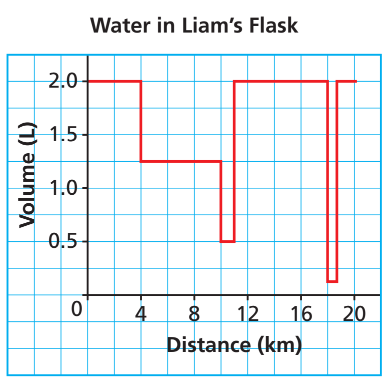
5.3

6. a) Laura cycles home from school, then walks back to school. Which graph best matches this situation? Explain your choice.



b) Choose one of the graphs in part a that did not describe Laura's journey. Describe a possible situation for the graph.

7. This graph shows the volume of water in Liam's flask as he hikes the Trans Canada trail.



a) Describe what is happening for each line segment of the graph.
b) How many times did Liam fill his flask?

c) How much water was in Liam's flask at the start of his hike?

d) Identify the dependent and independent variables.

5.4

8. The data below show how the temperature of boiling water as it cools is related to time.

a) Graph the data. Did you join the points? Why or why not?

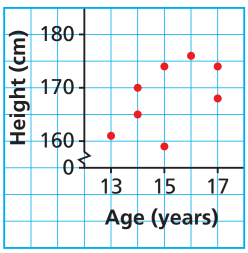
b) Does the graph represent a function? How can you tell?

Time (min)	Temperature (°C)
0	89
5	78
10	69
15	62
20	57
25	53
30	50

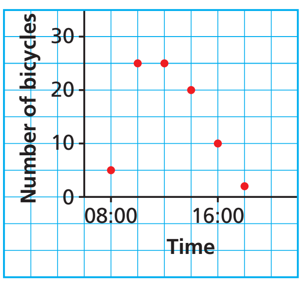
5.5

9. Which of these graphs represents a function? Justify your answer.
Write the domain and range for each graph.

a) **Heights and Ages of 8 Students**



b) **Number of Bicycles at School**



10. For the graphs below:

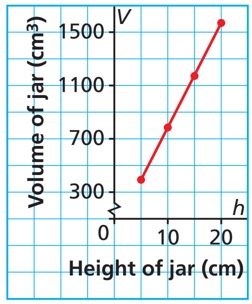
a) What does each graph represent?

b) Identify the independent and dependent variables.

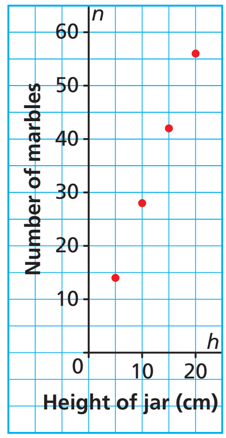
c) Write the domain and range for each graph. Estimate when necessary. Are there any restrictions on the domain and range? Explain.

d) Why are the points joined on one graph but not on the other?

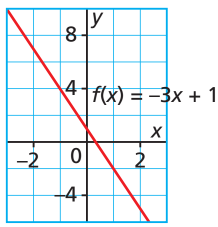
i) **Graph A**



ii) **Graph B**



11. This is a graph of the function $f(x) = -3x + 1$.



a) Determine the range value when the domain value is 1.

b) Determine the domain value when the range value is 4.

12. Sketch a graph of a function that has each domain and range.

a) domain: $-1 \leq x \leq 5$; range: $0 \leq y \leq 3$

b) domain: $x \leq 1$; range: $-2 \leq y \leq 2$

5.6

13. Which sets of ordered pairs represent linear relations? Explain your answers.

a) $\{(1, 5), (5, 5), (9, 5), (13, 5)\}$

b) $\{(1, 2), (1, 4), (1, 6), (1, 8)\}$

c) $\{(-2, -3), (-1, -2), (2, 1), (4, -3)\}$

14. **a)** For each equation, create a table of values when necessary, then graph the relation.

i) $x = 3$

ii) $y = 2x^2 + 3$

iii) $y = 2x + 3$

iv) $y = 3$

v) $y = 3x$

vi) $x + y = 3$

b) Which equations in part a represent linear relations? How do you know?

15. Isabelle manages her diabetes by taking insulin to control her blood sugar. The number of units of insulin taken, N, is given by the equation $N = \frac{1}{15}g$, where g represents the number of grams of carbohydrates consumed.

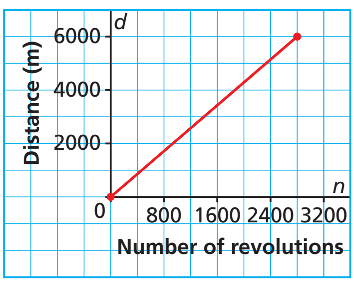
a) Explain why the equation represents a linear relation.

b) State the rate of change. What does it represent?

5.7

16. This graph shows the distance, d metres, travelled by Jadan on her bicycle as a function of the number of wheel revolutions, n, as she rode from Whitehorse to the Grey Mountain Road lookout in the Yukon.

Jadan's Bicycle Trip



a) How far was Jadan from the lookout when she started her bicycle trip?

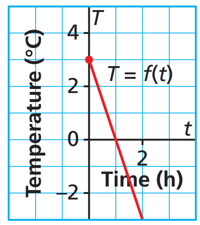
b) Write the domain and range.

c) Determine the rate of change. What does it represent?

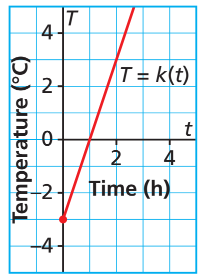
d) Use your answer to part c to determine the diameter of a bicycle wheel.

17. These graphs show the temperature, T degrees Celsius, as a function of time, t hours. Match each graph with its vertical intercept and rate of change.

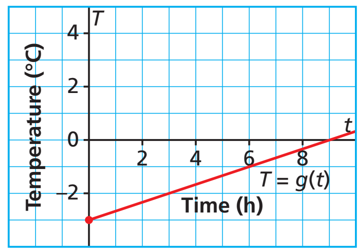
a)



b)



c)

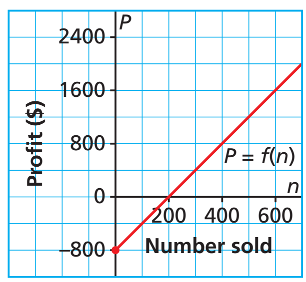


i) $-3°C; \frac{1}{3}°C/h$

ii) $3°C; -3°C/h$

iii) $-3°C; 3°C/h$

18. This graph shows the profit, P dollars, on a company's sale of n baseball caps.



a) How many baseball caps have to be sold before the company begins to make a profit?

b) What is the profit on the sale of each baseball cap?

c) How many caps have to be sold to make each profit?

i) $600 **ii)** $1200

d) In part c, when the profit doubles why does the number of baseballs caps sold not double?

PRACTICE TEST

For questions 1 and 2, choose the correct answer: A, B, C, or D

1. For the function $f(x) = 3 - 6x$, what is the value of $f(-3)$?

A. 1 **B.** 21 **C.** -15 **D.** 0

2. Which equation does *not* represent a linear function?

A. $f(x) = 5$ **B.** $f(x) = 5x$ **C.** $f(x) = 5x^2$ **D.** $f(x) = -5$

3. For each relation represented below:

i) State whether it is a function and how you know.

ii) If the relation is a function:
State its domain and range.
Represent the function in a different way.
State whether it is a linear function and how you know.

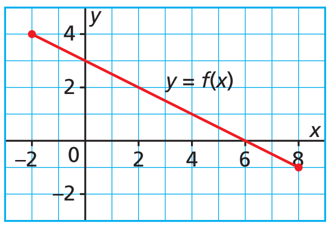
iii) If the relation is a linear function:
Identify the dependent and independent variables.
Determine the rate of change.

a) {(2, 5), (–3, 6), (1, 5), (–1, 4), (0, 2)}

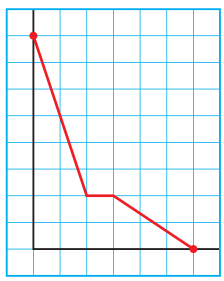
b)

n	s
2	4
-1	1
1	1
-3	9

c)



4. Describe a possible situation for this graph.
Label the axes and give the graph a title.
Justify your description.



5. This table of values shows how the time to cook a turkey is related to its mass.

a) Why is this relation a function?

b) Identify the dependent and the independent variables. Justify your choice.

c) Graph the data. Did you connect the points? Explain.

d) Determine the domain and range of the graph. Could you extend the graph? Identify and explain any restrictions on the domain and range. Explain.

e) Determine the rate of change for this function. What does it represent?

f) For how long should you cook a turkey with mass 7 kg?

Mass (kg)	Time (h)
4	2.5
6	3.0
8	3.5
10	4.0