5 Relations and Functions

BUILDING ON

- writing equations to represent patterns in tables
- graphing and analyzing linear relations

BIG IDEAS

- A relation associates the elements of one set with the elements of another set.
- A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.
- A linear function has a constant rate of change and its graph is a non-vertical straight line.

NEW VOCABULARY

relation
arrow diagram
function
domain
range
function notation
rate of change
linear function
vertical intercept
horizontal intercept

PAULATUUQ (place of coal) is in the Northwest Territories, north of the Arctic Circle. Here, on June 21st, the longest day of the year, the sun never sets.

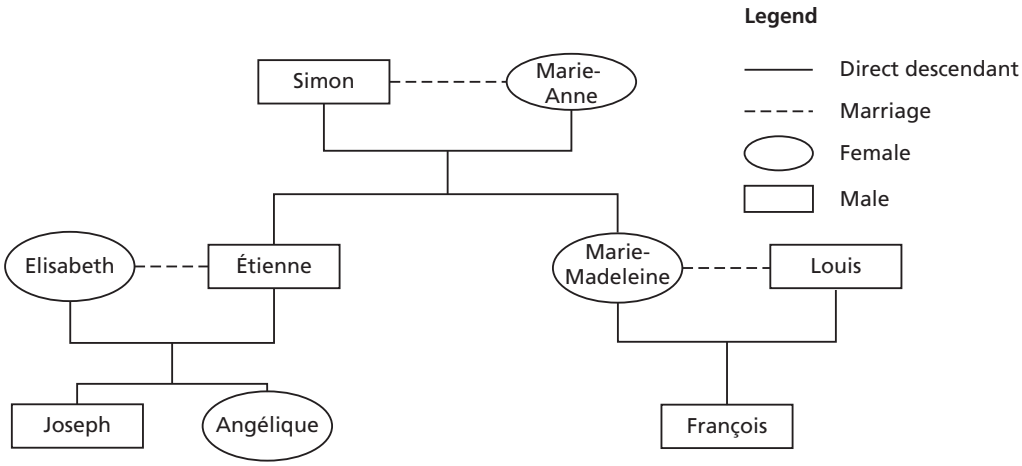
5.1 Representing Relations

LESSON FOCUS

Represent relations in different ways.

Make Connections

This family tree shows relations within a family.



How is Joseph related to Simon?

How are Angélique and François related?

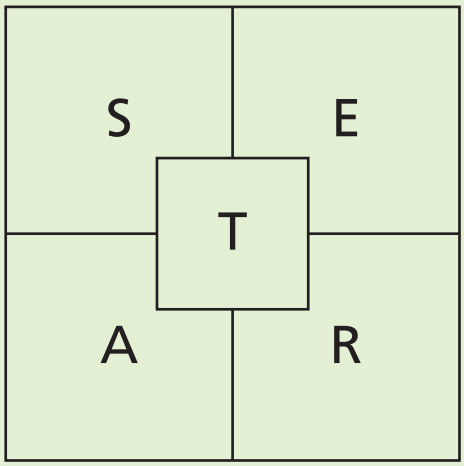
How does the family tree show these relations?

Construct Understanding

TRY THIS

A. Use the letters in the square below to write as many words as you can with 3 or more letters.

- Each word must contain the letter in the centre square.
- A letter can only be used once in each word.
- Plurals cannot be used.



B. Use the letters in the square and the words you wrote.
Pair a letter with a word to show the association "is a letter of the word." Write as many pairs as you can.

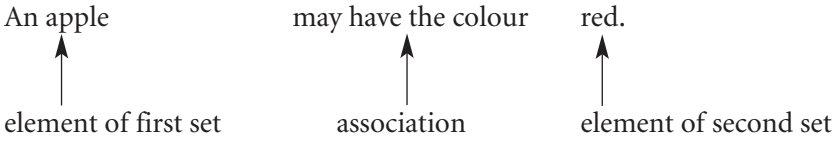
C. Use the same association. Suppose you reverse the order of the items in the pairs. Does the association make sense? Explain.

A *set* is a collection of distinct objects.
An *element* of a set is one object in the set.
A **relation** associates the elements of one set with the elements of another set.

One way to write a set is to list its elements inside braces.
For example, we can write the set of natural numbers from 1 to 5 as:
{1, 2, 3, 4, 5}

The order of the elements in the set does not matter.

Consider the set of fruits and the set of colours.
We can associate fruits with their colours.
For example:



What other relations could you create using the same association between fruits and colours?

So, this set of ordered pairs is a relation:

{(apple, red), (apple, green), (blueberry, blue), (cherry, red), (huckleberry, blue)}

Here are some other ways to represent this relation:

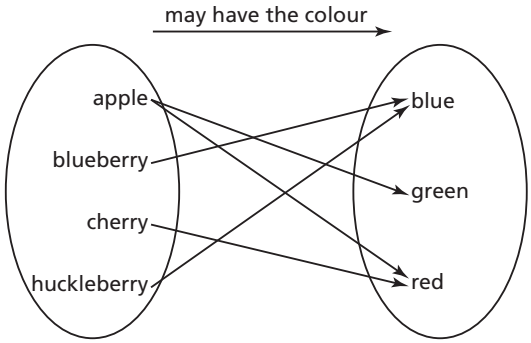
- a table

Fruit	Colour
apple	red
apple	green
blueberry	blue
cherry	red
huckleberry	blue

The heading of each column describes each set.

- an **arrow diagram**

The two ovals represent the sets.
Each arrow associates an element of the first set with an element of the second set.



The order of the words in the ordered pairs, the columns in the table, and the ovals in the arrow diagram is important. It makes sense to say, "an apple may have the colour red," but it makes no sense to say, "red may have the colour apple." That is, a relation has direction from one set to the other set.

Example 1 Representing a Relation Given as a Table

Northern communities can be associated with the territories they are in. Consider the relation represented by this table.

Community	Territory
Hay River	NWT
Iqaluit	Nunavut
Nanisivik	Nunavut
Old Crow	Yukon
Whitehorse	Yukon
Yellowknife	NWT

a) Describe this relation in words.

b) Represent this relation:
i) as a set of ordered pairs
ii) as an arrow diagram

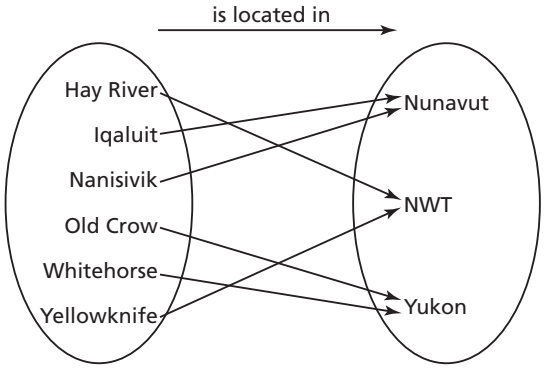
SOLUTION

a) The relation shows the association "is located in" from a set of northern communities to a set of territories. For example, Hay River is located in the NWT.

b) i) The communities are the first elements in the ordered pairs. The territories are the second elements in the ordered pairs. The ordered pairs are: {(Hay River, NWT), (Iqaluit, Nunavut), (Nanisivik, Nunavut), (Old Crow, Yukon), (Whitehorse, Yukon), (Yellowknife, NWT)}

ii) The communities are written in the first set of the arrow diagram.

The territories are written in the second set; each territory is written only once.



CHECK YOUR UNDERSTANDING

1. Animals can be associated with the classes they are in.

Animal	Class
ant	Insecta
eagle	Aves
snake	Reptilia
turtle	Reptilia
whale	Mammalia

a) Describe this relation in words.

b) Represent this relation:
i) as a set of ordered pairs
ii) as an arrow diagram

[Answers: a) The relation shows the association "belongs to the class" between a set of animals and a set of classes. b) i) {(ant, Insecta), (eagle, Aves), (snake, Reptilia), (turtle, Reptilia), (whale, Mammalia)}]

Why is the direction of the arrows in the arrow diagram important?

How are the representations the same?

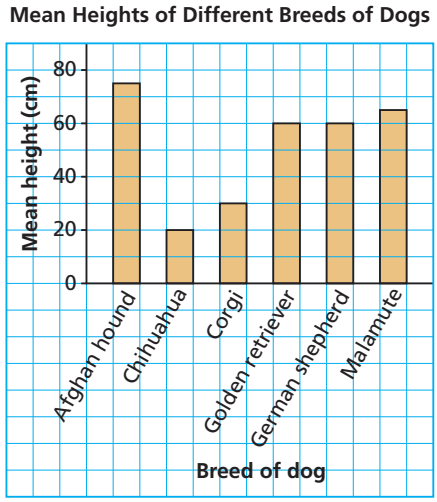
When the elements of one or both sets in a relation are numbers, the relation can be represented as a bar graph.

Example 2 Representing a Relation Given as a Bar Graph

Different breeds of dogs can be associated with their mean heights. Consider the relation represented by this graph. Represent the relation:

a) as a table

b) as an arrow diagram



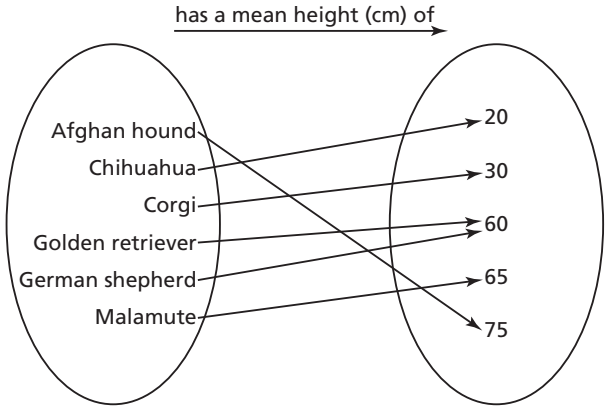
SOLUTION

a) The association is: "has a mean height of"

In the table, write the breeds of dogs in the first column and the mean heights in centimetres in the second column.

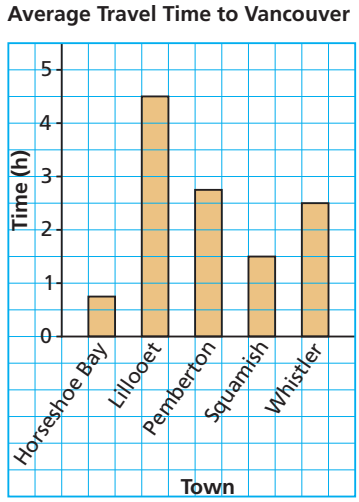
Breed of Dog	Mean Height (cm)
Afghan hound	75
Chihuahua	20
Corgi	30
Golden retriever	60
German shepherd	60
Malamute	65

b) In the arrow diagram, write the breeds of dogs in the first set and the mean heights in centimetres in the second set.



CHECK YOUR UNDERSTANDING

2. Different towns in British Columbia can be associated with the average time, in hours, that it takes to drive to Vancouver. Consider the relation represented by this graph.



Represent the relation:

a) as a table

b) as an arrow diagram

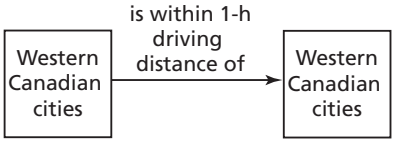
Answer: a)

Town	Average Time (h)
Horseshoe Bay	0.75
Lillooet	4.5
Pemberton	2.75
Squamish	1.5
Whistler	2.5

Sometimes a relation contains so many ordered pairs that it is impossible to list all of them or to represent them in a table.

Example 3 Identifying a Relation from a Diagram

In this diagram:



Both sets in a relation can be the same.

a) Describe the relation in words.

b) List 2 ordered pairs that belong to the relation.

SOLUTION

a) The relation shows the association "is within 1-h driving distance" from a set of cities in Western Canada to a set of cities in Western Canada.

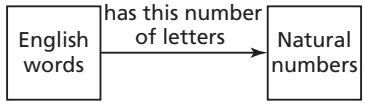
b) Two ordered pairs that belong to the relation are:
(Innisfail AB, Olds AB), (Kelowna B.C., Vernon B.C.)

CHECK YOUR UNDERSTANDING

3. In the diagram below:

a) Describe the relation in words.

b) List 2 ordered pairs that belong to the relation.



[Sample Answers: a) The relation shows the association "has this number of letters" from a set of English words to a set of natural numbers. b) (mathematics, 11) and (language, 8)]

Discuss the Ideas

1. What are the advantages and disadvantages of the different ways you can represent a relation?

2. Why is the order of the elements in an ordered pair important? Give an example.

Exercises

A

3. For each table below:

i) Describe the relation in words.

ii) Represent the relation:

- as a set of ordered pairs
- as an arrow diagram

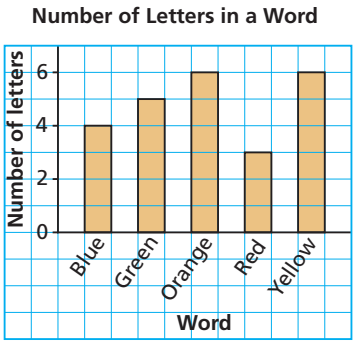
a)

Coin	Value ($)
penny	0.01
nickel	0.05
dime	0.10
quarter	0.25
loonie	1.00
toonie	2.00

b)

Sport	Equipment
badminton	shuttlecock
badminton	racquet
hockey	puck
hockey	stick
tennis	ball
tennis	racquet
soccer	ball

4. Consider the relation represented by this graph.



Represent the relation:

a) as a table

b) as an arrow diagram

B

5. This table shows some of Manitoba's francophone artists and the medium they use.

Artist	Medium
Gaëtanne Sylvester	sculpture
Hubert Théroux	painting
Huguette Gauthier	stained glass
James Culleton	painting
Nathalie Dupont	photography
Simone Hébert Allard	photography

a) Describe the relation in words.

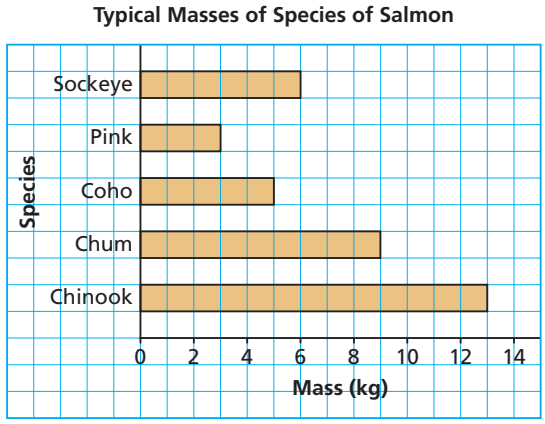
b) Represent this relation:

i) as a set of ordered pairs

ii) as an arrow diagram

Burning Sunset Detail by James Culleton

6. **a)** Describe the relation represented by this bar graph.

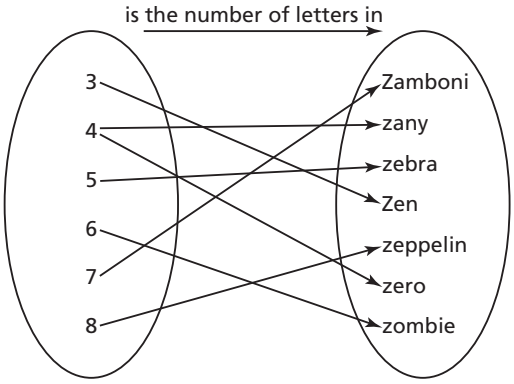


b) Represent the relation as a set of ordered pairs.

c) Represent the relation in a different way.

7. For a word game, words that begin with the letter Z can be difficult to find.

a) What does this arrow diagram represent?



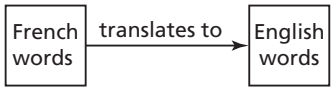
b) Represent this relation in two different ways.

c) Create an arrow diagram for words beginning with the letter X, then represent the relation in two different ways.

8. In the diagram below:

a) Describe the relation in words.

b) List two ordered pairs that belong to the relation.



9. A digital clock displays digits from 0 to 9 by lighting up different segments in two squares. For example, the digit 2 needs 5 segments to light up, as shown.

a) List the set of ordered pairs of the form: (digit, number of segments lit up)

b) Represent this relation in two different ways.

10. Here are some Canadian hockey players and the year they were born.

Jennifer Botterill (1979); Jonathan Cheechoo (1980); Roberto Luongo (1979); Jordin Tootoo (1983); Hayley Wickenheiser (1978)

For each association below, use these data to represent a relation in different ways.

a) was born in

b) is the birth year of

11. Choose five people in your class.

a) Use the association "is older than" to write a relation. Represent the relation using a set of ordered pairs.

b) Create your own association for these five people, then describe the relation in words. Represent this relation in different ways.

C

12. Two dice are rolled and the numbers that show are recorded.

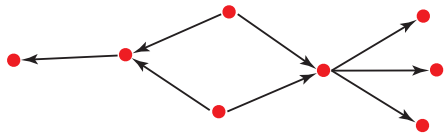
a) Use each association below to create a relation as a set of ordered pairs.

i) The sum of the numbers is even.

ii) The difference between the numbers is a prime number.

b) In part a, does the order of the numbers in each ordered pair matter? Explain.

13. The association "is the parent of" is shown in the diagram. Each dot represents a person and each arrow maps a parent to her or his child.



In this relation:

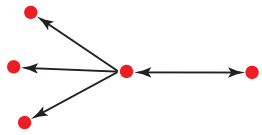
a) How many children are shown?

b) How many parents are shown?

c) How many grandparents are shown?

Justify your answers.

14. The association "is the sister of" is shown in the diagram. Each dot represents a person and each arrow maps a sister to a sibling.



In this relation:

a) How many females are shown?

b) How many males are shown?

Justify your answers.

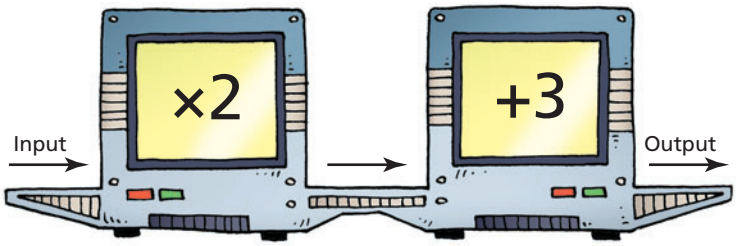
Reflect

Create a relation that you can describe in words. Show two different ways to represent your relation.

5.2 Properties of Functions

LESSON FOCUS

Develop the concept of a function.



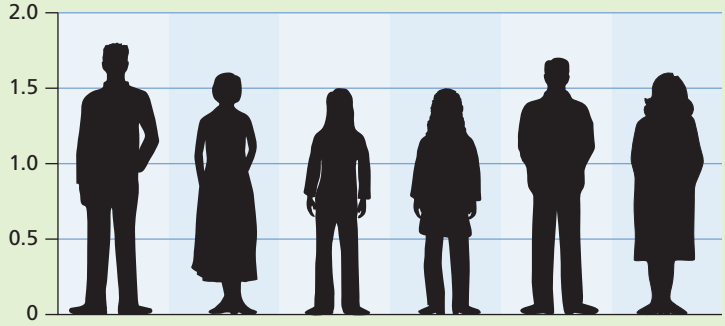
Make Connections

What is the rule for the Input/Output machine above?
Which numbers would complete this table for the machine?

Input	Output
1	5
2	7
	9
4	
	13

Construct Understanding

THINK ABOUT IT



	David	Gina	Sara	Emily	Jack	Helen
Height (m)	1.8	1.6	1.5	1.5	1.7	1.6
Age (years)	40	38	12	10	65	67

Work in a group of 3.
Use the picture above.

Each of you chooses one of the relations below.

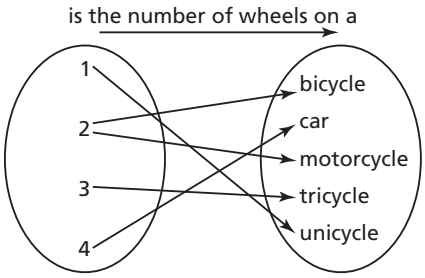
- *name* related to *age*
- *name* related to *height*
- *height* related to *name*

Represent the relation you chose. Compare the relations.
How are they alike? How are they different?

The set of first elements of a relation is called the **domain.**
The set of related second elements of a relation is called the **range.**
A **function** is a special type of relation where each element in the domain is associated with exactly one element in the range.

Here are some different ways to relate vehicles and the number of wheels each has.

This relation associates a number with a vehicle with that number of wheels.

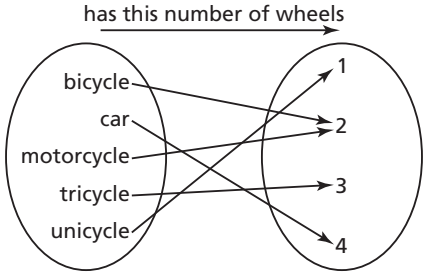


This diagram does not represent a function because there is one element in the first set that associates with two elements in the second set; that is, there are two arrows from 2 in the first set.

{(1, unicycle), (2, bicycle), (2, motorcycle), (3, tricycle), (4, car)}

The set of ordered pairs above does not represent a function because two ordered pairs have the same first element.
The domain is the set of first elements: {1, 2, 3, 4}
The range is the set of associated second elements: {unicycle, bicycle, motorcycle, tricycle, car}

This relation associates a vehicle with the number of wheels it has.



This diagram represents a function because each element in the first set associates with exactly one element in the second set; that is, there is only one arrow from each element in the first set.

{(unicycle, 1), (bicycle, 2), (motorcycle, 2), (tricycle, 3), (car, 4)}

The set of ordered pairs above represents a function because the ordered pairs have different first elements.
The domain is the set of first elements: {unicycle, bicycle, motorcycle, tricycle, car}
The range is the set of associated second elements: {1, 2, 3, 4}

When we list the elements of the range, we do not repeat an element that occurs more than once.

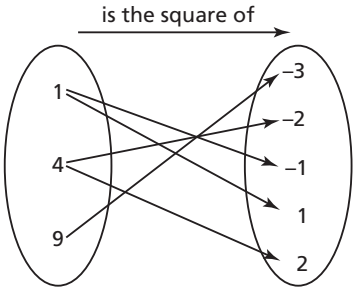
Example 1 Identifying Functions

For each relation below:

- Determine whether the relation is a function. Justify the answer.
- Identify the domain and range of each relation that is a function.

a) A relation that associates given shapes with the number of right angles in the shape: {(right triangle, 1), (acute triangle, 0), (square, 4), (rectangle, 4), (regular hexagon, 0)}

b)

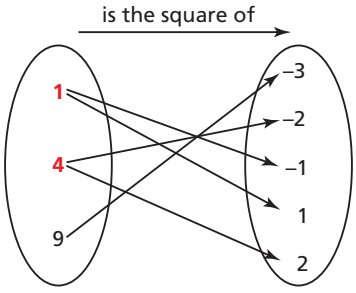


SOLUTION

a) Check to see if any ordered pairs have the same first element: {(right triangle, 1), (acute triangle, 0), (square, 4), (rectangle, 4), (regular hexagon, 0)}
Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.

The domain is the set of the first elements of the ordered pairs: {right triangle, acute triangle, square, rectangle, regular hexagon}
The range is the set of second elements of the ordered pairs: {0, 1, 4}

b) Check to see if any element in the first set associates with more than one element in the second set.



This relation is not a function because each of the numbers 1 and 4 in the first set associates with more than one number in the second set.

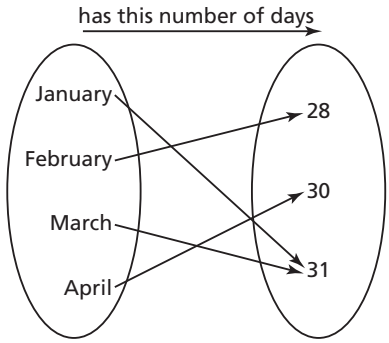
CHECK YOUR UNDERSTANDING

1. For each relation below:

- Determine whether the relation is a function. Justify your answer.
- Identify the domain and range of each relation that is a function.

a) A relation that associates a number with a prime factor of the number: {(4, 2), (6, 2), (6, 3), (8, 2), (9, 3)}

b)



[Answers: a) no b) yes; domain: {January, February, March, April}; range: {28, 30, 31}]

What other strategies could you use to determine whether each relation is a function?

If (a, b) and (a, c) are ordered pairs in a function, what can you say about b and c?

In the workplace, a person's gross pay, P dollars, often depends on the number of hours worked, h.
So, we say P is the *dependent variable*. Since the number of hours worked, h, does not depend on the gross pay, P, we say that h is the *independent variable*.

independent variable →

Hours Worked, h	Gross Pay, P ($)
1	12
2	24
3	36
4	48
5	60

← dependent variable

domain

range

A table of values usually represents a sample of the ordered pairs in a relation.

The values of the independent variable are listed in the first column of a table of values. These elements belong to the domain.

The values of the dependent variable are listed in the second column of a table of values. These elements belong to the range.

Example 2 Describing Functions

The table shows the masses, m grams, of different numbers of identical marbles, n.

Number of Marbles, n	Mass of Marbles, m (g)
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

a) Why is this relation also a function?

b) Identify the independent variable and the dependent variable. Justify the choices.

c) Write the domain and range.

(Solution continues.)

CHECK YOUR UNDERSTANDING

2. The table shows the costs of student bus tickets, C dollars, for different numbers of tickets, n.

Number of Tickets, n	Cost, C ($)
1	1.75
2	3.50
3	5.25
4	7.00
5	8.75

a) Why is this relation also a function?

b) Identify the independent variable and the dependent variable. Justify your choices.

c) Write the domain and range.

[Answers: b) n; C c) $\{1, 2, 3, 4, 5, \ldots\}$; $\{1.75, 3.50, 5.25, 7.00, 8.75, \ldots\}$]

SOLUTION

a) For each number in the first column, there is only one number in the second column. So, the relation is a function.

b) From an understanding of the situation, the mass of the marbles, m, depends on the number of marbles, n. So, m is the dependent variable and n is the independent variable.

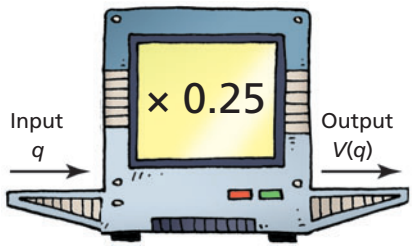
c) The first column of the table is representative of the domain.
The domain is: {1, 2, 3, 4, 5, 6,...}; that is, all natural numbers

The second column of the table is representative of the range.
The range is: {1.27, 2.54, 3.81, 5.08, 6.35, 7.62,...}; that is, the product of each natural number and 1.27

We can think of a function as an input/output machine. The input can be any number in the domain, and the output depends on the input number. So, the input is the independent variable and the output is the dependent variable.

Consider two machines that both accept quarters. Machine A calculates the value of the quarters. Machine B weighs the quarters. Each machine performs a different operation, so the machines represent two different functions.

- Machine A



Function V

When the input is q quarters, the output or value, V, in dollars is: $0.25q$
The equation $V = 0.25q$ describes this function.

Since V is a function of q, we can write this equation using **function notation**:
$V(q) = 0.25q$
We say: "V of q is equal to $0.25q$."

This notation shows that V is the dependent variable and that V depends on q.

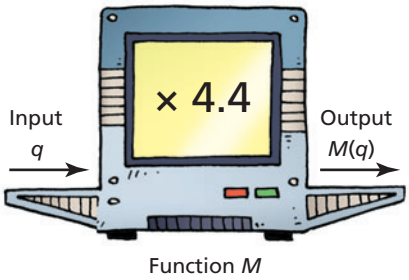
$V(3)$ represents the value of the function when $q = 3$.

$V(3) = 0.25(3)$

$V(3) = 0.75$

So, the value of 3 quarters is \$0.75.

- Machine B



The mass of 1 quarter is 4.4 g.

When the input is q quarters, the output or mass, M, in grams is: $4.4q$
The equation $M = 4.4q$ describes this function.
Since M is a function of q, we can write this equation using function notation:
$M(q) = 4.4q$
This notation shows that M is the dependent variable and that M depends on q.

Any function that can be written as an equation in two variables can be written in function notation. For example, to write the equation $d = 4t + 5$ in function notation, we may write $d(t) = 4t + 5$. t represents an element of the domain and $d(t)$ represents an element of the range.

When we write an equation that is not related to a context, we use x as the independent variable and y as the dependent variable. Then an equation in two variables such as $y = 3x - 2$ may be written as $f(x) = 3x - 2$.

We can use any other letter, such as g or h or k, to name a function.

Conversely, we may write an equation in function notation as an equation in two variables.
For example, for the equation $C(n) = 300 + 25n$, we write $C = 300 + 25n$.
And, for the equation $g(x) = -2x + 5$, we write $y = -2x + 5$.

Example 3 Using Function Notation to Determine Values

The equation $V = -0.08d + 50$ represents the volume, V litres, of gas remaining in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it is empty.

a) Describe the function.
Write the equation in function notation.

b) Determine the value of $V(600)$.
What does this number represent?

c) Determine the value of d when $V(d) = 26$.
What does this number represent?

(Solution continues.)

CHECK YOUR UNDERSTANDING

3. The equation $C = 25n + 1000$ represents the cost, C dollars, for a feast following an Arctic sports competition, where n is the number of people attending.

a) Describe the function.
Write the equation in function notation.

(Question continues.)

SOLUTION

a) The volume of gas remaining in a vehicle's tank is a function of the distance travelled. In function notation:
$V(d) = -0.08d + 50$

b) To determine $V(600)$, use:

$V(d) = -0.08d + 50$ Substitute: $d = 600$
$V(600) = -0.08(600) + 50$
$V(600) = -48 + 50$
$V(600) = 2$

$V(600)$ is the value of V when $d = 600$.
This means that when the car has travelled 600 km, the volume of gas remaining in the vehicle's tank is 2 L.

c) To determine the value of d when $V(d) = 26$, use:

$V(d) = -0.08d + 50$ Substitute: $V(d) = 26$
$26 = -0.08d + 50$ Solve for d.
$-24 = -0.08d$ Divide each side by -0.08.
$d = 300$

$V(300) = 26$ means that when $d = 300$, $V = 26$; that is, after the car has travelled 300 km, 26 L of gas remains in the vehicle's tank.

b) Determine the value of $C(100)$. What does this number represent?

c) Determine the value of n when $C(n) = 5000$. What does this number represent?

[Answers: a) $C(n) = 25n + 1000$ b) \$3500 c) 160]

What values of d do not make sense as possible domain values?

Discuss the Ideas

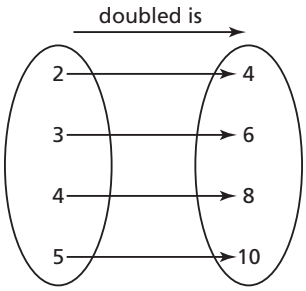
1. How can you tell whether a set of ordered pairs represents a function?
2. When a function is completely represented using a set of ordered pairs or a table of values, how can you determine the domain and range of the function?
3. Why are some relations not functions? Why are all functions also relations?

Exercises

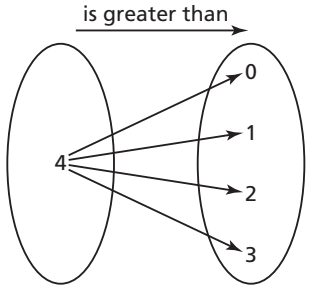
A

4. Which arrow diagrams represent functions?

a)



b)



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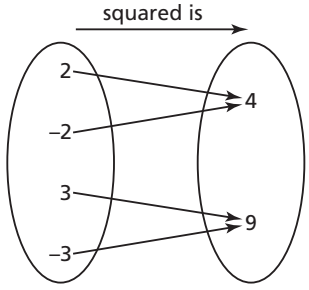


5. Which sets of ordered pairs represent functions? Identify the domain and range of each set of ordered pairs.

a) $\{(1, 3), (2, 6), (3, 9), (4, 12)\}$

b) $\{(1, 0), (0, 1), (-1, 0), (0, -1)\}$

c) $\{(2, 3), (4, 5), (6, 7), (8, 9)\}$

d) $\{(0, 1), (0, 2), (1, 2), (0, 3), (1, 3), (2, 3)\}$

6. Write in function notation.

a) $C = 20n + 8$

b) $P = n - 3$

c) $t = 5d$

d) $y = -x$

7. Write as an equation in two variables.

a) $d(t) = 3t - 5$

b) $f(x) = -6x + 4$

c) $C(n) = 5n$

d) $P(n) = 2n - 7$

B

8. For each relation below:

- Determine whether the relation is a function. Justify your answer.
- Identify the domain and range of each relation.

a) $\{(1, 1), (2, 8), (3, 27), (4, 64)\}$

b) $\{(3, 4), (3, 5), (3, 6), (3, 7)\}$

9. For each table of values below:

i) Explain why the relation is a function.

ii) Identify the independent variable and the dependent variable. Justify your choices.

iii) Write the domain and range.

a)

Number of Cans of Juice Purchased, n	Cost, C ($)
1	2.39
2	4.00
3	6.39
4	8.00
5	10.39
6	12.00

b)

Altitude, A (m)	Temperature, T (°C)
610	15.0
1220	11.1
1830	7.1
2440	3.1
3050	−0.8
3660	−4.8

10. This set of ordered pairs associates a number with a polygon that has that number of sides:
{(3, isosceles triangle), (3, equilateral triangle), (3, right triangle), (3, scalene triangle), (4, square), (4, rectangle), (4, rhombus), (4, trapezoid), (4, parallelogram), (5, pentagon), (6, hexagon)}

a) Does the set of ordered pairs represent a function? Explain.

b) Suppose the elements in the ordered pairs were reversed. Use the association "has this number of sides." Would the new relation be a function? Explain.

c) Identify the domain and range of each relation in parts a and b.

11. The Rassemblement jeunesse francophone in Alberta brings together French language high school students from all over the province for a day of activities. Use two columns in this table to represent a relation.

a) Name two relations that are functions.

b) Name two relations that are not functions.

Justify your answers.

Name	From	Age	Gender
Marie	Edmonton	13	F
Gabriel	Falher	16	M
Élise	Bonnyville	14	F
Christophe	Calgary	13	M
Jean	Edmonton	15	M
Mélanie	Edmonton	15	F
Nicole	Red Deer	17	F
Marc	Légal	13	M

12. Which statement is true? Give an example to justify your choice.

a) All functions are relations, but not all relations are functions.

b) All relations are functions, but not all functions are relations.

13. In a crossword game, each letter is worth a certain number of points. Here are some letters and their points.

a) Create two different tables to represent relations that associate these letters and their points.

b) Which table in part a represents a function? Justify your choice.

14. For the function $f(x) = -5x + 11$, determine:

a) $f(1)$ **b)** $f(-3)$

c) $f(0)$ **d)** $f(1.2)$

15. **a)** For the function $f(n) = 2n - 7$, determine n when:

i) $f(n) = 11$ **ii)** $f(n) = -6$

b) For the function $g(x) = -5x + 1$, determine x when:

i) $g(x) = 41$ **ii)** $g(x) = -16$

16. The function $C(i) = 2.54i$ converts a measurement of i inches to a measurement of C centimetres.

a) Write the function as an equation in 2 variables.

b) Determine the value of $C(12)$. What does this number represent?

c) Determine the value of i when $C(i) = 100$. What does this number represent?

17. A car is travelling toward Meadow Lake Park, Saskatchewan. The equation $D = -80t + 300$ represents the distance, D kilometres, to Meadow Lake after t hours of driving.

a) Describe the function.
Write this equation in function notation.

b) How far away from Meadow Lake Park was the car at the start of its journey?
How do you know?

18. Anthropologists who study human remains have developed equations for estimating the height of a person from a measure of her or his bones. The height in centimetres is a function of the length, l centimetres, of the humerus (the upper arm bone).

For a female: $f(l) = 2.754l + 71.475$
For a male: $m(l) = 2.894l + 70.641$

a) Determine each value. What does each number represent?

i) $f(15)$ **ii)** $m(20)$

b) Determine each value of l. What does each number represent?

i) $f(l) = 142$ **ii)** $m(l) = 194$

c) Measure the length of your humerus. Use an equation to estimate your height. How close was your answer to your actual height?

19. The function $C(f) = \frac{5}{9}(f - 32)$ converts a temperature, f degrees Fahrenheit, to C degrees Celsius.

a) Determine:

i) $C(50)$ **ii)** $C(-13)$

b) Determine each value of f when:

i) $C(f) = 20$ **ii)** $C(f) = -35$

c) Write an equation in function notation to relate the temperatures in each fact.

i) Pure water freezes at 0°C or 32°F.

ii) Pure water boils at 100°C or 212°F.

iii) Cookies are baked at 180°C or 356°F.

C

20. To convert a temperature in degrees Celsius to degrees Fahrenheit, multiply the Celsius temperature by $\frac{9}{5}$ then add 32. Use these instructions to write an equation in function notation for this conversion.

21. The area of a rectangle with length l centimetres and width w centimetres is 9 cm^2. Express the perimeter of the rectangle as a function of its length.

22. A rectangle with length l centimetres and width w centimetres has a perimeter of 12 cm. Use function notation to express the length of the rectangle as a function of its width. What are the domain and range of the function?

23. The lengths of the sides of a triangle, in units, are s, $s + 5$, and t. Its perimeter is 16 units. Use function notation to express t as a function of s. What are the domain and range of the function?

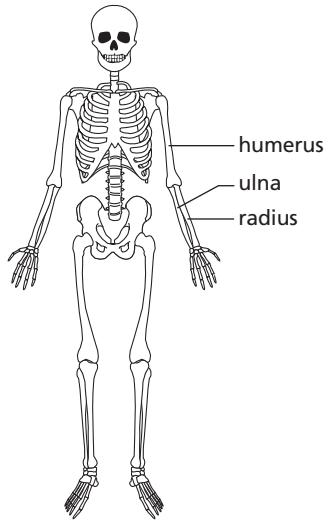
Reflect

Describe how you can determine if a relation is a function. Use an example to illustrate each strategy you might use.

THE WORLD OF MATH

Careers: Forensic Anthropologist

Forensic anthropologists study human remains to understand more about how people develop, both as individuals and as societies. They collect data on bones and teeth to identify the sex, height, mass, race, and age at death. Forensic anthropologists work in crime labs, law enforcement agencies, museums, or at archaeological sites; and may give expert testimony in court. They may identify bones and bone fragments that have been in storage for many years.



Specimens must be cleaned, accurately measured, and catalogued. Measurements of the skull and teeth can help to estimate the age of a person. When the length of the humerus, radius, or ulna is known, then a person's height can be approximated.

Here are some typical equations used to estimate the height of a person from the length of her or his radius.

For a female: $h = 3.343r + 81.224$
For a male: $h = 3.271r + 89.925$,

where h is the height of the person in centimetres and r is the length of the radius in centimetres.

CHECKPOINT 1

Connections

Here is a Frayer model for a function.

Definition

A function is a relation where each element in the first set is associated with exactly one element in the second set.

Essential Characteristics

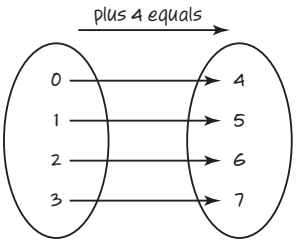
The domain is the set of first elements in the ordered pairs. These are the values of the independent variable. The range is the set of second elements in the ordered pairs. These are the values of the dependent variable.

Function

Example

x	y
0	4
1	5
2	6
3	7

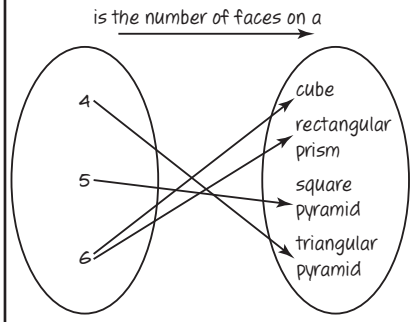
{(0, 4), (1, 5), (2, 6), (3, 7)}



Non-examples

Number of Faces	Object
4	triangular pyramid
5	square pyramid
6	cube
6	rectangular prism

{(4, triangular pyramid),
(5, square pyramid),
(6, cube),
(6, rectangular prism)}



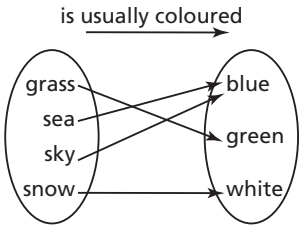
Concept Development

- **In Lesson 5.1**
  - You described a relation in words and represented it using: a set of ordered pairs, an arrow diagram, a table, and a bar graph.
- **In Lesson 5.2**
  - You identified a function by checking to see whether its ordered pairs had different first elements.
  - You listed the elements of the domain and of the range.
  - You related the elements of the domain to the independent variable and the elements of the range to the dependent variable.
  - You described functions in words, and algebraically using function notation.

Assess Your Understanding

5.1

1. Copy and complete this table for different representations of relations.

	Description in Words	Set of Ordered Pairs	Arrow Diagram	Table or Graph
a)		{(skin, drum), (skin, kayak), (bark, basket), (stone, inukshuk), (stone, carving)}		
b)				**Number** / **Number of Factors**: 1 / 1; 2 / 2; 3 / 2; 4 / 3
c)				
d)	For the numbers 1 to 4, the first number in an ordered pair is greater than the second number.			

5.2

2. **a)** Which relations in question 1 are functions? Justify your answers.
b) State the domain and range of each function.

3. **a)** Think about two sets of numbers and an association.
i) Create a relation that is not a function.
ii) Create a function.
b) Represent each relation in part a in different ways.

4. The temperature, T degrees Celsius, of Earth's interior is a function of the distance, d kilometres, below the surface: $T(d) = 10d + 20$
a) Identify the dependent and independent variables.
b) Write this function as an equation in two variables.
c) Determine the value of $T(5)$. Describe what this number represents.
d) Determine the value of d when $T(d) = 50$. Describe what this number represents.

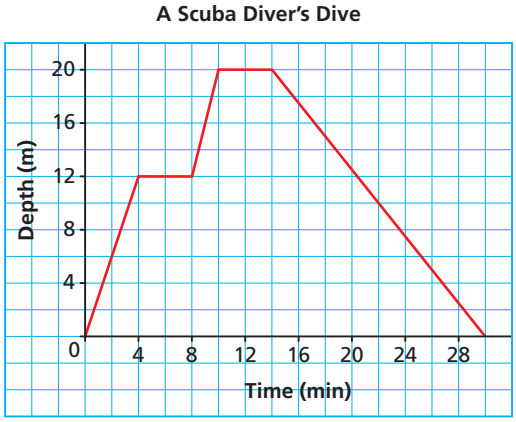
5.3 Interpreting and Sketching Graphs

LESSON FOCUS

Describe a possible situation for a given graph and sketch a possible graph for a given situation.

Make Connections

In math, a graph provides much information.
This graph shows the depth of a scuba diver as a function of time.



How many minutes did the dive last?

At what times did the diver stop her descent?

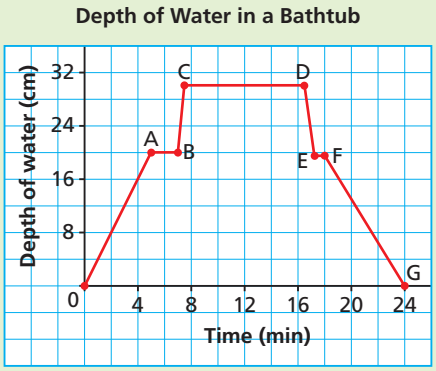
What was the greatest depth the diver reached? For how many minutes was the diver at that depth?

Construct Understanding

TRY THIS

Work with a partner.
You will need grid paper.

This graph shows the depth of water in a bathtub as a function of time.

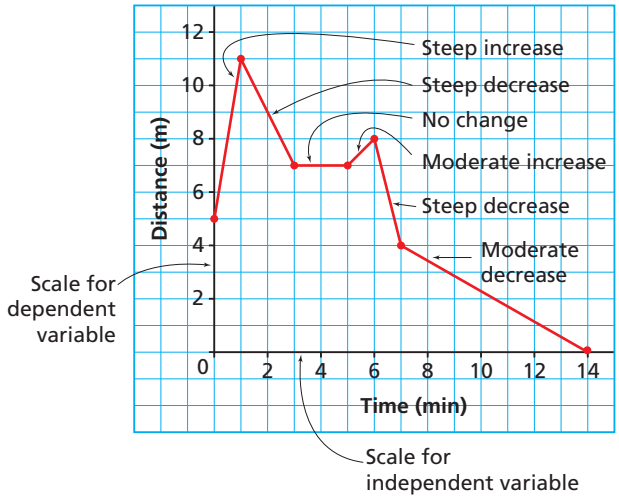


A. What does each segment of the graph represent? Compare your description with that of your partner. Are both your stories the same? Should they be? Explain.

B. Sketch a graph to represent this situation:
You put the plug in the bath and turn on the taps.
You leave the bathroom and return to discover that the bath has overflowed.
You turn off the taps and pull out the plug to let out some water. You put the plug back in.

C. Compare your graph with that of your partner. How are the graphs the same? How are they different?

The properties of a graph can provide information about a given situation.



Example 1 Interpreting a Graph

Each point on this graph represents a bag of popping corn. Explain the answer to each question below.

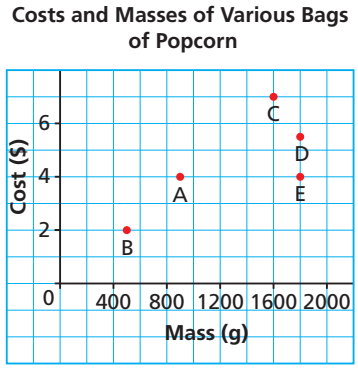
a) Which bag is the most expensive? What does it cost?

b) Which bag has the least mass? What is this mass?

c) Which bags have the same mass? What is this mass?

d) Which bags cost the same? What is this cost?

e) Which of bags C or D has the better value for money?



SOLUTION

a) Bag C is most expensive because it is represented by the highest point on the graph and the vertical axis represents cost. It costs $7.00.

b) Bag B has the least mass because it is represented by the point on the graph farthest to the left and the horizontal axis represents mass. The mass appears to be 500 g.

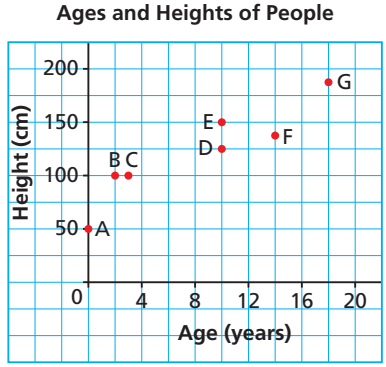
c) Bags D and E have the same mass because the points that represent them lie on the same vertical line and it passes through 1800 on the *Mass* axis. The mass is 1800 g.

d) Bags A and E cost the same because the points that represent them lie on the same horizontal line and it passes through 4 on the *Cost* axis. The cost is $4.00.

e) Bag D has the better value for money because it has a greater mass than bag C and costs less than bag C.

CHECK YOUR UNDERSTANDING

1. Each point on this graph represents a person. Explain your answer to each question below.



a) Which person is the oldest? What is her or his age?

b) Which person is the youngest? What is her or his age?

c) Which two people have the same height? What is this height?

d) Which two people have the same age? What is this age?

e) Which of person B or C is taller for her or his age?

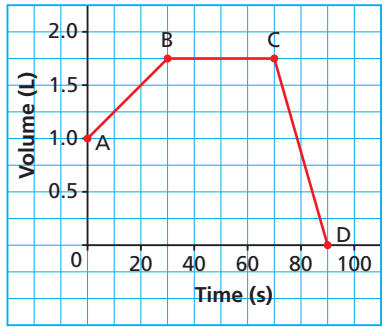
[Answers: a) G, 18 years
b) A, newborn
c) B and C, 100 cm
d) D and E, 10 years
e) B]

Does this graph represent a function? Explain.

Why do you think bag D is more expensive than bag E?

The graph shows how the volume of water in a watering can changes over time.
The starting volume is 1 L, which is the volume at point A.
Segment AB goes up to the right, so the volume of water is increasing from 0 s to 30 s.
Segment BC is horizontal, so the volume is constant from 30 s to 70 s.
Segment CD goes down to the right, so the volume is decreasing from 70 s to 90 s.
At point D, the volume is 0 L after 90 s.

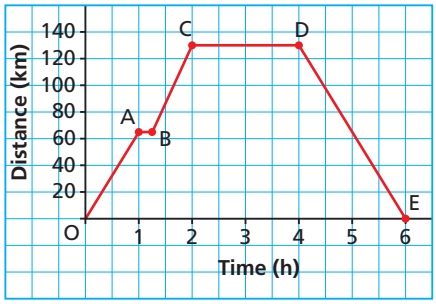
Volume of Water in a Watering Can



Example 2 Describing a Possible Situation for a Graph

Describe the journey for each segment of the graph.

Day Trip from Winnipeg to Winkler, Manitoba



The distance between Winnipeg and Winkler is 130 km.

SOLUTION

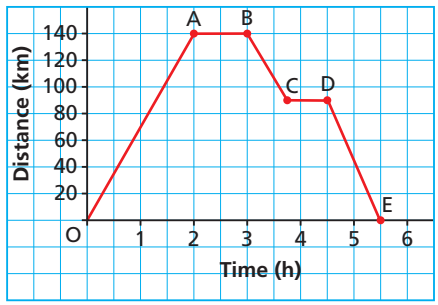
Segment	Graph	Journey
OA	The graph goes up to the right, so as time increases, the distance from Winnipeg increases.	In the first hour, the car leaves Winnipeg and travels approximately 65 km toward Winkler.
AB	The graph is horizontal, so as time increases, the distance stays the same.	The car stops for approximately 15 min.
BC	The graph goes up to the right, so as time increases, the distance increases.	The car travels approximately 65 km toward Winkler.
CD	The graph is horizontal, so as time increases, the distance stays the same.	At C, the car has travelled 130 km so it has reached Winkler, where it stops for 2 h.
DE	The graph goes down to the right, so as time increases, the distance decreases.	The car returns to Winnipeg and takes 2 h to travel 130 km.

CHECK YOUR UNDERSTANDING

2. This graph represents a day trip from Athabasca to Kikino in Alberta, a distance of approximately 140 km.

Describe the journey for each segment of the graph.

Day Trip from Athabasca to Kikino



[Answer: The car takes 2 h to travel 140 km to Kikino; the car stops for 1 h: the car takes approximately 45 min to travel 50 km toward Athabasca; the car stops for approximately 45 min; the car takes 1 h to travel approximately 90 km to Athabasca]

What was the total driving time? Explain.

What are the dependent and the independent variables?

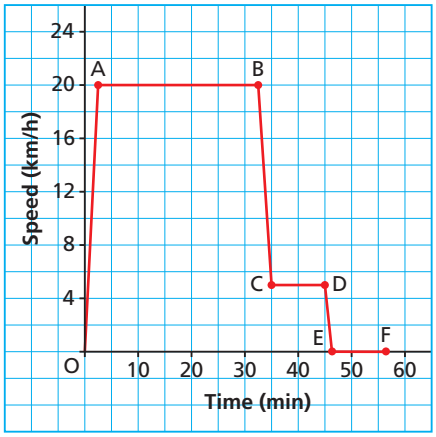
Example 3 Sketching a Graph for a Given Situation

Samuel went on a bicycle ride. He accelerated until he reached a speed of 20 km/h, then he cycled for 30 min at approximately 20 km/h. Samuel arrived at the bottom of a hill, and his speed decreased to approximately 5 km/h for 10 min as he cycled up the hill. He stopped at the top of the hill for 10 min.

Sketch a graph of speed as a function of time. Label each section of the graph, and explain what it represents.

SOLUTION

Draw and label axes on a grid. The horizontal axis represents time in minutes. The vertical axis represents speed in kilometres per hour.

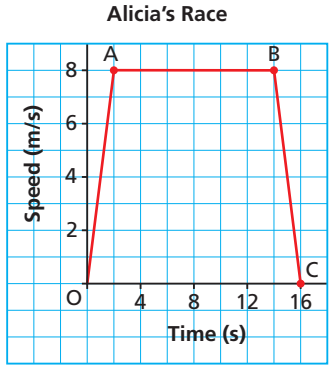


Segment	Journey
OA	Samuel's speed increases from 0 to 20 km/h, so the segment goes up to the right.
AB	Samuel cycles at approximately 20 km/h for 30 min. His speed does not change, so the segment is horizontal.
BC	Samuel's speed decreases to 5 km/h, so the segment goes down to the right.
CD	Samuel cycles uphill at approximately 5 km/h for 10 min. His speed does not change, so the segment is horizontal.
DE	Samuel slows down to 0 km/h, so his speed decreases and the segment goes down to the right.
EF	Samuel remains stopped at 0 km/h for 10 min, so the segment is horizontal.

CHECK YOUR UNDERSTANDING

3. At the beginning of a race, Alicia took 2 s to reach a speed of 8 m/s. She ran at approximately 8 m/s for 12 s, then slowed down to a stop in 2 s. Sketch a graph of speed as a function of time. Label each section of your graph, and explain what it represents.

Answer:



Why does the graph not end at point E?

Discuss the Ideas

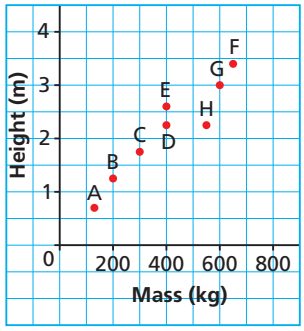
1. For a graph of distance as a function of time, what does each segment represent?
   - a horizontal line segment
   - a segment that goes up to the right
   - a segment that goes down to the right
2. For a graph of speed as a function of time, what does each segment represent?
   - a horizontal line segment
   - a segment that goes up to the right
   - a segment that goes down to the right

Exercises

A

3. Each point on the graph represents a polar bear. Explain the answer to each question below.

Heights and Masses of 8 Polar Bears

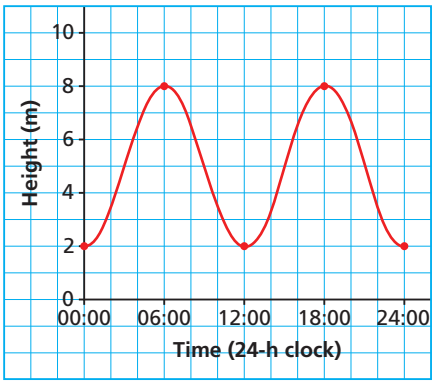


a) Which bear has the greatest mass? What is this mass?
b) Which bear is the shortest? What is its height?
c) Which two bears have the same mass? What is this mass?
d) Which two bears have the same height? What is this height?

B

4. This graph shows the height of the tide in a harbour as a function of time in one day. Explain the answer to each question below.

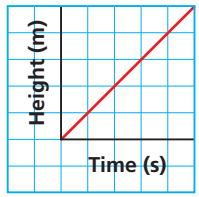
Height of the Tide in a Harbour



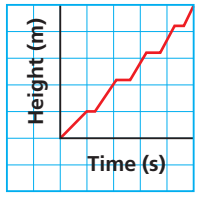
a) What is the greatest height? At what times does it occur?
b) What is the least height? At what times does it occur?
c) How high is the tide at 04:00?
d) When is the tide 4 m high?

5. To raise a flag, Sepideh pulls the rope steadily with both hands for a short time, then moves both hands up the rope and pulls again. She does this until the flag has been raised. Which graph best represents the height of the flag? Give reasons for your choice.

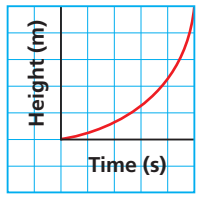
Graph A



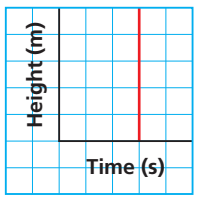
Graph B



Graph C

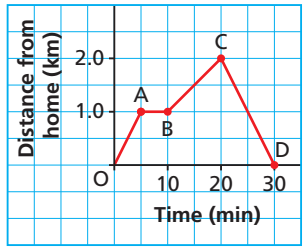


Graph D

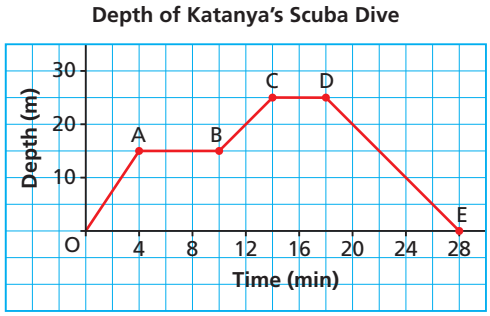


6. Gill runs for exercise. This graph shows her distance from home during one of her runs. Describe Gill's run for each segment of the graph.

Gill's Run



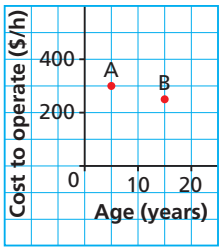
7. Katanya went scuba diving in Egypt. This graph shows her depth below sea level as a function of time on one of her dives.



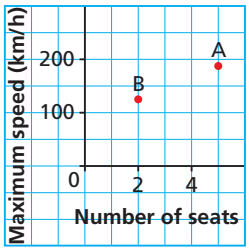
Write all that you know about the dive from the graph.

8. Point A and Point B represent the same helicopters in each of these graphs.

Graph of Cost against Age



Graph of Maximum Speed against Number of Seats



Which statements are true? Justify your answers.

a) The older helicopter is cheaper to operate.

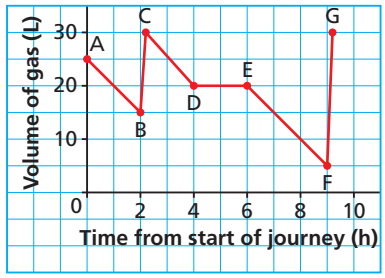
b) The helicopter with more seats has the lower maximum speed.

c) The helicopter with the lower maximum speed is cheaper to operate.

d) The helicopter with the greater maximum speed is older.

e) The helicopter with fewer seats is newer.

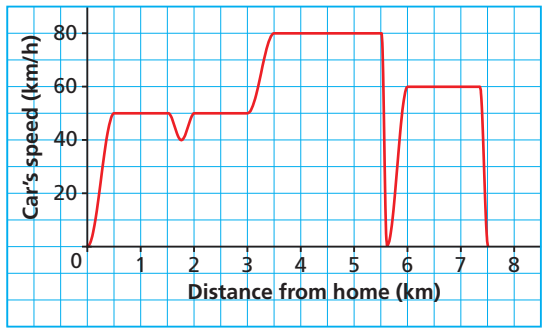
9. **a)** Describe what is happening for each line segment in this graph.



b) How much gas was in the tank at the start of the journey? Was the tank full at this time? Explain.

10. An oven is turned on at a room temperature of 20°C and it takes 10 min to reach a temperature of 190°C. A tray of cookies is placed in the oven to bake for 10 min. The oven is then turned off and returns to room temperature after 15 min. Sketch a graph of temperature as a function of time. Label each section of the graph and explain what it represents.

11. Write all that you know about a person's drive to work from this graph.



12. A school vending machine sells cartons of milk and juice. On a typical day:

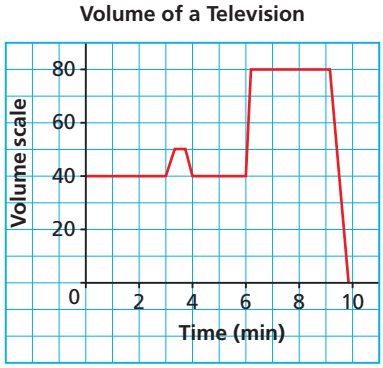
- No cartons are sold between 7 A.M. and 8 A.M., or from 5 P.M. onward.
- The machine has a capacity of 100 cartons. At 7 A.M., it is three-quarters full.
- From 8 A.M. to 10 A.M., 10:15 A.M. to noon, and from 1 P.M. to 3 P.M. the students are in class.
- The machine is filled at 11 A.M. and at 4 P.M.

Sketch a graph of the number of cartons in the vending machine as a function of time. Explain what each section of the graph represents.

13. A student drew a graph to represent this situation.

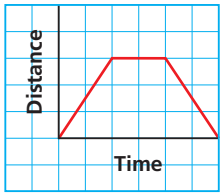
"Jonah is watching television. After 3 min his mom enters the room to ask him a question. He turns the volume down a bit, answers his mom, then turns the volume back up. Two minutes later, Jonah's dad turns on the dishwasher so Jonah gradually turns up the volume. After a further 3 min, a commercial comes on so Jonah presses the mute button."

Describe any errors in the student's graph.

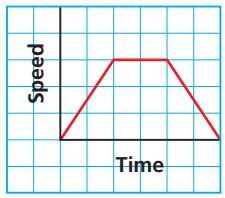


14. The two graphs below have the same shape, but different vertical axes. Copy each graph and include numbers and units on both axes. Write and justify a possible situation that it represents.

a) Graph A

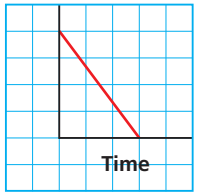


b) Graph B

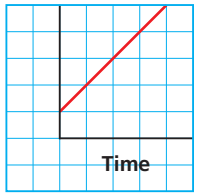


15. Each graph shows a quantity as a function of time. Choose a dependent variable for each graph, and suggest a possible situation that it represents. Copy the graph and include numbers on the axes.

a)



b)



C

16. Chad goes bungee jumping.

a) Sketch each graph of his jump.

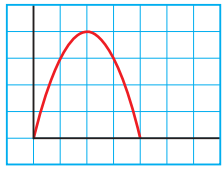
i) the distance above the ground as a function of time

ii) the speed as a function of time

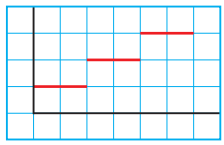
b) Explain the similarities and differences between the two graphs.

17. For each graph, choose a dependent variable and an independent variable, and suggest a possible situation that it represents. Describe the significance of any key points or changes in the graph.

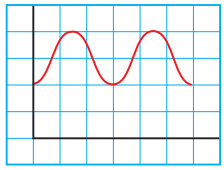
a)



b)

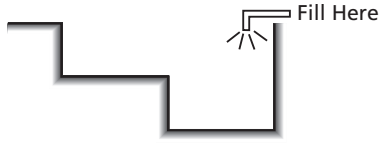


c)

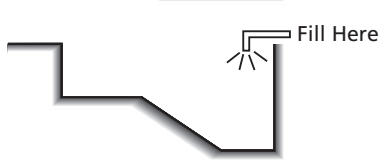


18. The diagrams below show cross-sections of swimming pools that will be filled with water at the same constant rate. Sketch two graphs on the same grid to represent the depth of water in each pool as a function of time. Label the axes. Justify the shape of each graph.

a)



b)



Reflect

When you describe a possible situation for a given graph, what features of the graph do you have to use in your description and how do you use them? Include a sketch of a graph in your explanation.

MATH LAB

5.4 Graphing Data

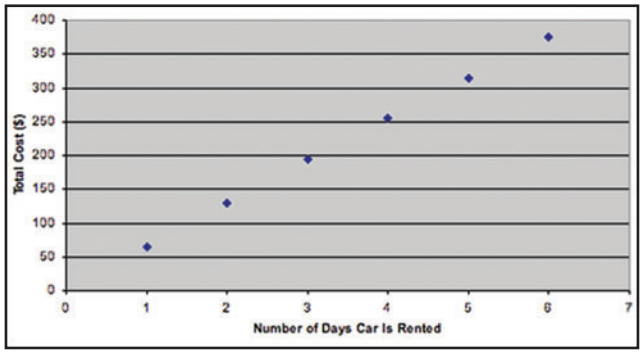
LESSON FOCUS

Graph data and investigate the domain and range when the data represent a function.

Make Connections

To rent a car for less than one week from Ace Car Rentals, the cost is $65 per day for the first three days, then $60 a day for each additional day.

Number of Days Car Is Rented	Total Cost ($)
1	65
2	130
3	195
4	255
5	315
6	375



Why are the points on the graph not joined?

Is this relation a function? How can you tell?

What is the domain? What is the range?

Construct Understanding

TRY THIS

Work with a partner.

You will need:

- a length of rope
- a metre stick
- grid paper, a graphing calculator, or a computer with graphing software

When you tie knots in a rope, the length of the rope is related to the number of knots tied.

A. You will investigate the relation between the number of knots and the length of rope.

- Measure the length of the rope without any knots. Tie a knot in the rope. Measure the length of the rope with the knot.
- Repeat the measurements for up to 5 knots. Try to tie the knots so they are all the same size and tightness.
- Record the number of knots and the length of the rope in a table.

Number of Knots	Length of Rope (cm)

B. Graph the data.

- How did you determine on which axis to plot each variable?
- How did you choose the scale for each axis so the data fit on the axes?
- Did you join the points? Justify your answer.

C. If you used a graphing calculator, sketch the graph. If you used a computer, print the graph.

- Is the length of the rope a function of the number of knots? Explain. If your answer is yes, list the set of ordered pairs. What is the domain? What is the range?
- Would it make sense to extend the graph to the right? To the left? If your answer is yes, how far could you extend it? What is the new domain? What is the new range?
  If your answer is no, what restrictions are there on the domain and range?

Suppose you used another piece of rope with the same length. Would the length of the rope after each number of knots be the same as that which you first recorded? Why or why not?

D. Suppose you combined your data with those of 4 pairs of classmates.

- When you graph all the data, does the graph represent a function? Justify your answer.
- Suppose you calculated the mean rope length for each number of knots, then graphed the data. Would the graph represent a function? Justify your answer.

Assess Your Understanding

1. For each table of values below:

i) Graph the data. Will you join the points? Justify your answer.

ii) Does the graph represent a function? Explain.

a) At a constant pressure, the speed of sound in air is related to the air temperature.

Air Temperature (°C)	Speed of Sound (m/s)
0	331
5	334
10	337
15	340
20	343

b) The recommended daily dose of vitamin C is related to a female's age in years.

Age (years)	Dose of Vitamin C Tablet (mg)
3	15
6	25
9	45
12	45
15	65
18	65
21	75

2. Graph the data in these tables of values from Lesson 5.2, question 9. Decide whether to join the points.

How can you tell from each graph that the relation is a function?

a)

Number of Cans of Juice Purchased, n	Cost, C ($)
1	2.39
2	4.00
3	6.39
4	8.00
5	10.39
6	12.00

b)

Altitude, A (m)	Temperature, T (°C)
610	15.0
1220	11.1
1830	7.1
2440	3.1
3050	−0.8
3660	−4.8

5.5 Graphs of Relations and Functions

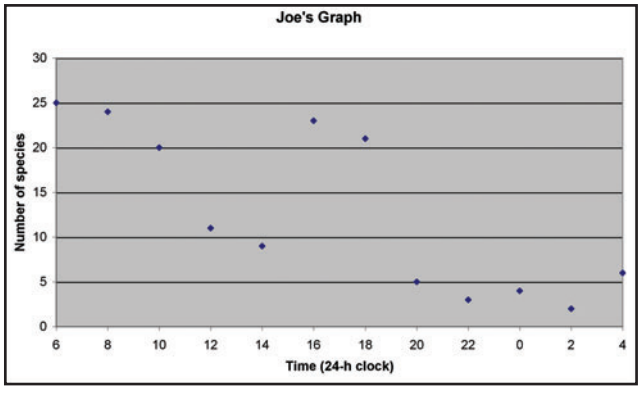
The great horned owl is Alberta's provincial bird.

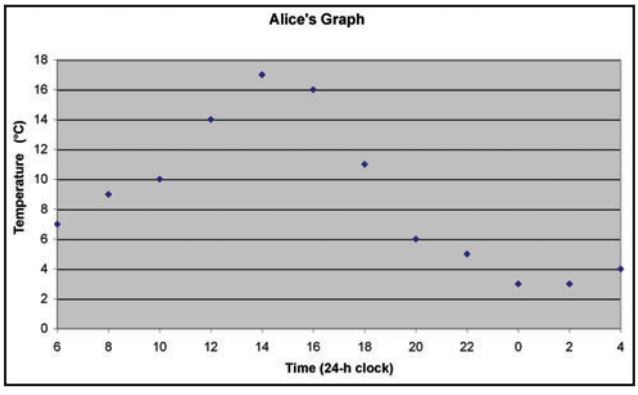
LESSON FOCUS

Determine the properties of the graphs of relations and functions.

Make Connections

In an environmental study in Northern Alberta, Joe collected data on the numbers of different species of birds he heard or saw in a 1-h period every 2 h for 24 h. Alice collected data on the temperature in the area at the end of each 1-h period. They plotted their data:





Does each graph represent a relation? A function? How can you tell?

Which of these graphs should have the data points connected? Explain.

Construct Understanding

TRY THIS

Work with a partner.

You will need grid paper.

A. Each of you chooses one of these tasks:

- A sugar cube has a volume of 5 cm^3 and a mass of 4 g. Graph the mass of sugar as a function of the number of sugar cubes from 0 to 5 sugar cubes.
- Five cubic centimetres of loose sugar also has a mass of 4 g. Graph the mass of sugar as a function of the volume of sugar from 0 to 25 cm^3 of loose sugar.

B. Share your results. How are your graphs alike? How are they different?

C. Work together:

- Identify the dependent variable and independent variable for each function. How did you decide on which axis to graph each variable?
- How did you decide whether to connect the points?
- Are there any restrictions on the domain and range? Explain.

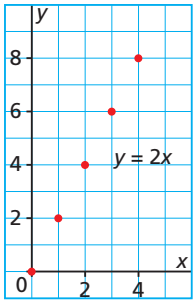
We can represent the function that associates every whole number with its double in several ways.

Using a table of values:

Whole Number, x	Double the Number, y
0	0
1	2
2	4
3	6
4	8

The table continues for all whole numbers.
The domain is the set of whole numbers.
The range is the set of even whole numbers.

Using a graph:



How would the graph change if both x and y were real numbers?

How can you tell the domain and range from the graph?

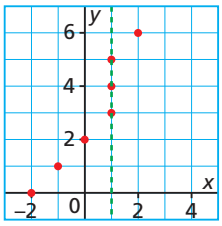
We know the relation $y = 2x$ is a function because each value of x associates with exactly one value of y, and each ordered pair has a different first element.

The *domain* of a function is the set of values of the independent variable; for the graph above, the domain is the x-values.

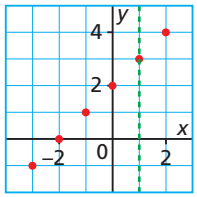
The *range* of a function is the set of values of the dependent variable; for the graph above, the range is the y-values.

When the domain is restricted to a set of discrete values, the points on the graph are not connected.

A relation that is not a function has two or more ordered pairs with the same first coordinate. So, when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point.



A function has ordered pairs with different first coordinates. So, when the ordered pairs of the function are plotted on a grid, any vertical line drawn will always pass through no more than one point.



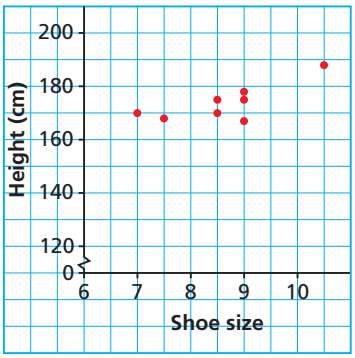
Vertical Line Test for a Function

A graph represents a function when no two points on the graph lie on the same vertical line.
Place a ruler vertically on a graph, then slide the ruler across the graph. If one edge of the ruler always intersects the graph at no more than one point, the graph represents a function.

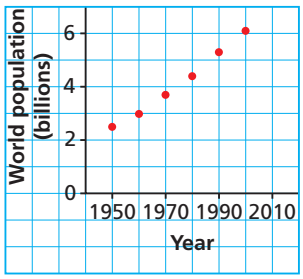
Example 1 Identifying whether a Graph Represents a Function

Which of these graphs represents a function? Justify the answer.

a) Height against Shoe Size



b) World Population

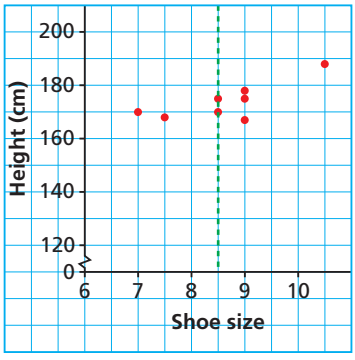


SOLUTION

Use the vertical line test for each graph.

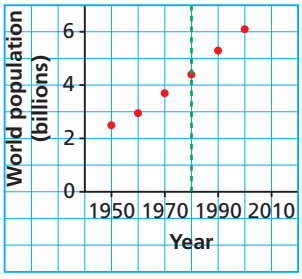
a) This graph does not represent a function because two points lie on the same vertical line.

Height against Shoe Size



b) This graph does represent a function.
Any vertical line drawn on the graph passes through 0 points or 1 point.

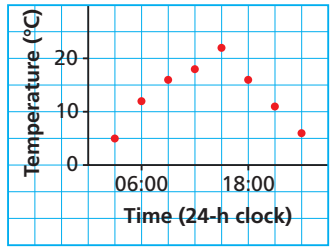
World Population



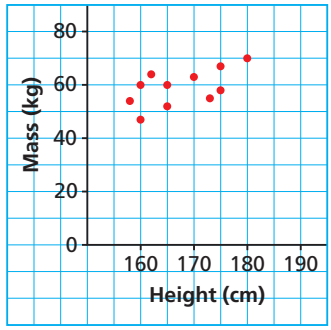
CHECK YOUR UNDERSTANDING

1. Which of these graphs represents a function? Justify your answer.

a) Outside Temperature over a 24-h Period



b) Masses of Students against Height



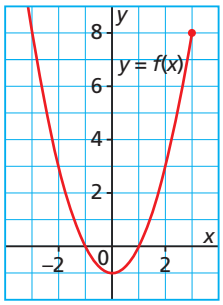
[Answers: a) function
b) not a function]

How does the vertical line test relate to the definition of a function?

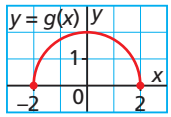
Example 2 Determining the Domain and Range of the Graph of a Function

Determine the domain and range of the graph of each function.

a)



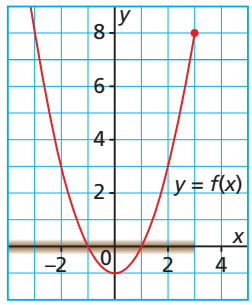
b)



SOLUTION

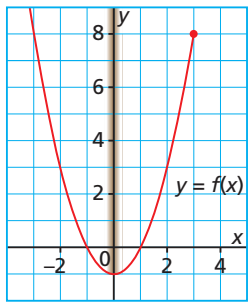
a) The dot at the right end of the graph indicates that the graph stops at that point.

There is no dot at the left end of the graph, so the graph continues to the left.

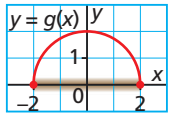


The domain is the set of x-values of the function.
Visualize the shadow of the graph on the x-axis.
The domain is the set of all real numbers less than or equal to 3; that is, $x \leq 3$.

The range is the set of y-values of the function.
Visualize the shadow of the graph on the y-axis.
The range is the set of all real numbers greater than or equal to -1; that is, $y \geq -1$.



b) The dot at each end of the graph indicates that the graph stops at that point.



The domain is the set of x-values of the function.
Visualize the shadow of the graph on the x-axis.
The domain is the set of real numbers between -2 and 2, including these numbers; that is, $-2 \leq x \leq 2$.

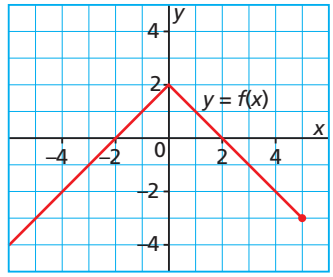
We say: "x is greater than or equal to -2 and less than or equal to 2."

(Solution continues.)

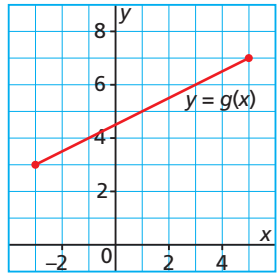
CHECK YOUR UNDERSTANDING

2. Determine the domain and range of the graph of each function.

a)



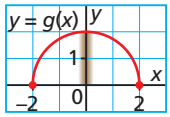
b)



[Answers: a) $x \leq 5$; $y \leq 2$
b) $-3 \leq x \leq 5$; $3 \leq y \leq 7$]

When data are not discrete, we use inequality symbols to indicate the domain and range.

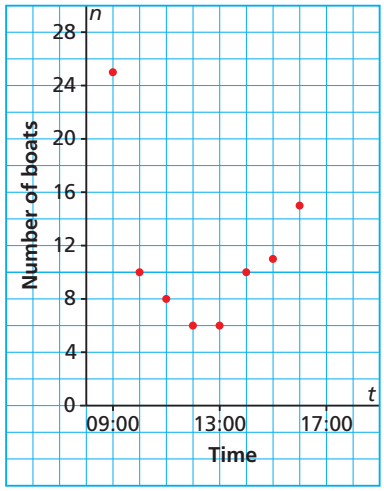
The range is the set of y-values of the function.
Visualize the shadow of the graph on the y-axis.
The range is the set of real numbers between 0 and 2, including these numbers; that is, $0 \leq y \leq 2$.



Example 3 Determining the Domain and Range of the Graph of a Situation

This graph shows the number of fishing boats, n, anchored in an inlet in the Queen Charlotte Islands as a function of time, t.

Number of Fishing Boats Anchored in an Inlet



a) Identify the dependent variable and the independent variable. Justify the choices.

b) Why are the points on the graph not connected? Explain.

c) Determine the domain and range of the graph.

SOLUTION

a) The number of fishing boats is a function of time. Since the number of boats, n, depends on the time of day, the dependent variable is n and the independent variable is t.

b) The points on the graph are not connected because the number of boats is restricted to a whole number. This means that most values between the points are not valid; for example, between 10:00 and 11:00, the number of boats decreases from 10 to 8. We may plot a point at $n = 9$, if we know a corresponding time, but no other point is valid between 10 and 8 because we cannot have a fractional number of boats.

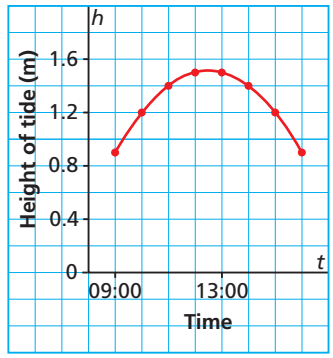
c) The domain is the set of times; that is,
{09:00, 10:00, 11:00, 12:00, 13:00, 14:00, 15:00, 16:00}

The range is the set of the numbers of boats; that is,
{6, 8, 10, 11, 15, 25}

CHECK YOUR UNDERSTANDING

3. This graph shows the approximate height of the tide, h metres, as a function of time, t, at Port Clements, Haida Gwaii on June 17, 2009.

Height of Tide at Port Clements, June 17, 2009



a) Identify the dependent variable and the independent variable. Justify your choices.

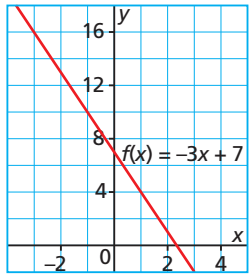
b) Why are the points on the graph connected? Explain.

c) Determine the domain and range of the graph.

[Answers: a) h, t
c) $09{:}00 \leq t \leq 16{:}00$;
$0.9 \leq h \leq 1.5$]

Example 4 Determining Domain Values and Range Values from the Graph of a Function

Here is a graph of the function $f(x) = -3x + 7$.



a) Determine the range value when the domain value is -2.

b) Determine the domain value when the range value is 4.

SOLUTION

The domain value is a value of x. The range value is a value of $f(x)$.

a) To determine the value of $f(x)$ when $x = -2$:

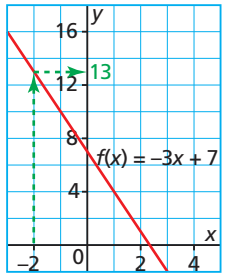
Begin at $x = -2$ on the x-axis.

Draw a vertical line to the graph, then a horizontal line to the y-axis.

The line appears to intersect the y-axis at 13.

So, $f(-2) = 13$

When the domain value is -2, the range value is 13.



b) To determine the value of x when $f(x) = 4$:

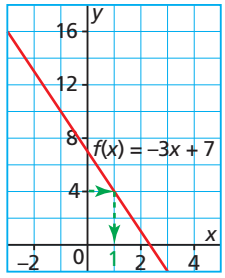
Since $y = f(x)$, begin at $y = 4$ on the y-axis.

Draw a horizontal line to the graph, then a vertical line to the x-axis.

The line intersects the x-axis at 1.

So, when $f(x) = 4$, $x = 1$

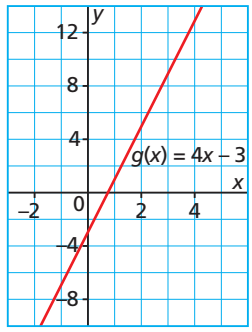
When the range value is 4, the domain value is 1.



When the domain value is -2, the range value is 13.

CHECK YOUR UNDERSTANDING

4. Here is a graph of the function $g(x) = 4x - 3$.



a) Determine the range value when the domain value is 3.

b) Determine the domain value when the range value is -7.

[Answers: a) 9 b) -1]

Discuss the Ideas

1. How do you decide whether to connect the points you plot for a graph?

2. What can you tell about the domain and range of a function from its graph?

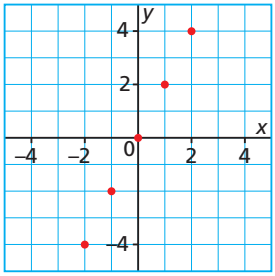
3. How can you identify whether a graph represents a function?

Exercises

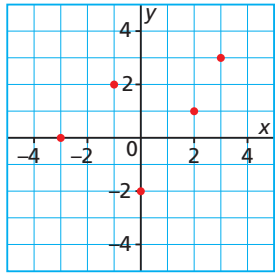
A

4. List the domain and the range of the graph of each function.

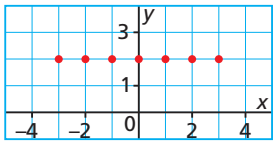
a)



b)



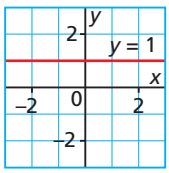
c)



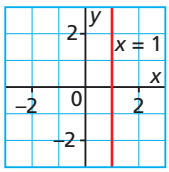
5. How can you tell that each graph in question 4 represents a function?

6. Which of these graphs represents a function? Justify your answer.

a)

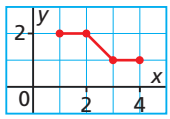


b)

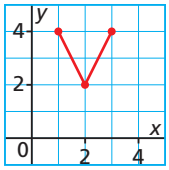


7. Match the graph of each function to its domain and range listed below.

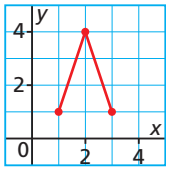
a)



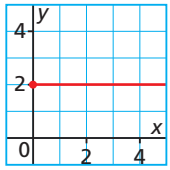
b)



c)



d)

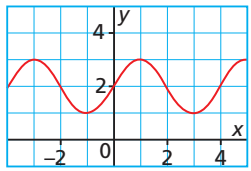


i) domain: $1 \leq x \leq 3$; range: $2 \leq y \leq 4$
ii) domain: $1 \leq x \leq 3$; range: $1 \leq y \leq 4$
iii) domain: $x \geq 0$; range: $y = 2$
iv) domain: $1 \leq x \leq 4$; range: $1 \leq y \leq 2$

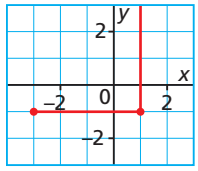
B

8. Which of these graphs represents a function? Justify your answer.
Write the domain and range for each graph.

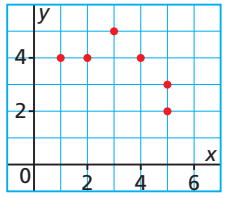
a)



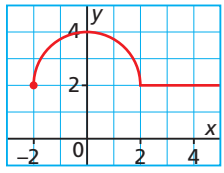
b)



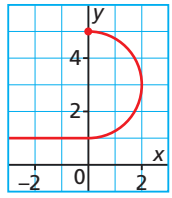
c)



d)

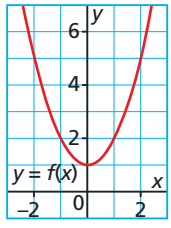


e)

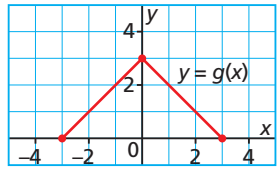


9. Determine the domain and range of the graph of each function.

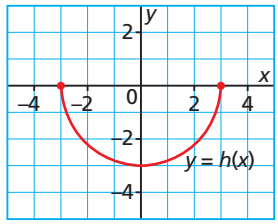
a)



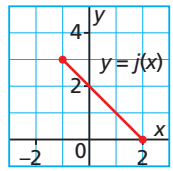
b)



c)



d)



10. Suppose a student drew a graph of each function described below. For which graphs should the student connect the points? Justify your answers.

a) The cost of a custom-made T-shirt is a function of the number of letters on the T-shirt.

b) The altitude of a plane is a function of the time it is in the air.

c) The mass of a baby is a function of her age.

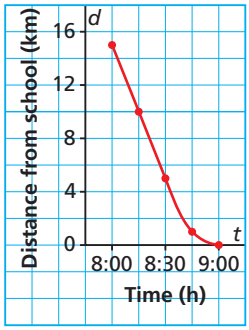
d) The cube root of a real number is a function of the number.

11. **a)** What do the data in each graph represent?

i)

Graph A

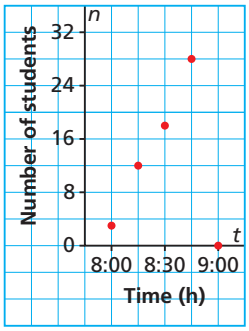
Distance of School Bus from School



ii)

Graph B

Number of Students on a School Bus

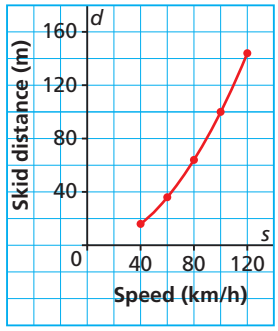


b) Identify the independent and dependent variables.

c) Why are the points connected on one graph but not on the other?

12. When police officers investigate a car crash, they can estimate the speed the car was travelling by measuring the skid distance.

Skid Distance of a Car

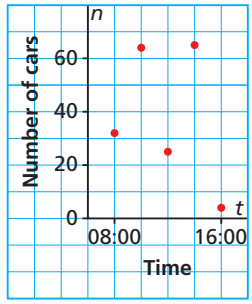


a) Why are the points on the graph connected?

b) Estimate the domain and range of the graph. Are there any restrictions on the domain and range? Explain.

13. This graph shows the number of cars, n, in the school parking lot as a function of time, t.

Number of Cars in the School Parking Lot



a) Identify the independent and dependent variables. Justify your choices.

b) Why are the points on the graph not connected?

c) Estimate the domain and range of the graph. Are there any restrictions on the domain and range? Explain.

14. Paulatuuq is north of the Arctic Circle. The table shows the number of hours, h, the sun is above the horizon every 60 days from January 1st, which is day 0.

Day	h
0	0
60	9.7
120	18.5
180	24.0
240	15.9
300	7.4
360	0

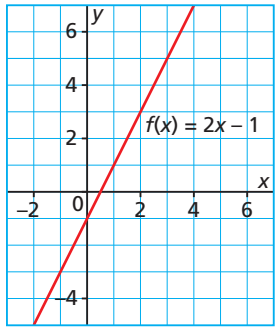
a) Identify the independent variable and the dependent variable. Justify your choices.
b) Graph the data in the table. Did you connect the points? Why or why not?
c) Use the table of values and the graph to explain why this relation is a function.

15. One litre of latex paint covers approximately 8.5 m^2 and costs \$12.
a) Copy and complete this table.

Volume of Paint, p (L)	0	2	4	6	8
Cost, c (\$)	0	24			
Area Covered, A (m^2)	0	17			

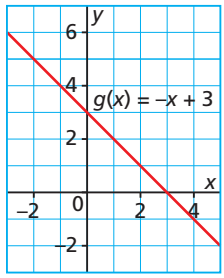
b) Graph the area covered as a function of the volume of paint.
c) Graph the area covered as a function of the cost.
d) Write the domain and range of the functions in parts b and c.

16. This is a graph of the function $f(x) = 2x - 1$.



a) Determine the range value when the domain value is 0.
b) Determine the domain value when the range value is 5.

17. This is a graph of the function $g(x) = -x + 3$.



a) Determine the range value when the domain value is -2.
b) Determine the domain value when the range value is 0.

18. Draw a graph of a function on a grid. Write the domain and range of the function. Exchange graphs with a classmate, and check that the domain and range of your classmate's graph are correct. If they are not, correct them, then explain your corrections to your classmate.

19. Sketch a graph of a function that has each domain and range.
a) domain: $-2 \leq x \leq 3$; range: $1 \leq y \leq 5$
b) domain: $x \geq 1$; range: $-1 \leq y \leq 1$

20. One planetary year is the time it takes for a planet to travel once around the sun. Since the planets take different times to travel around the sun, one year on each planet is different. The distance from Earth to the sun is 1 astronomical unit. Other distances in the solar system are compared to the distance from Earth to the sun.

	Earth	Jupiter	Saturn	Uranus
Distance from Sun (astronomical units)	1	5	10	19
Planetary Year (Earth years)	1	12	29	84

a) Graph planetary year as a function of distance from the sun. Did you connect the points? Explain.
b) Write the domain and range of this function.

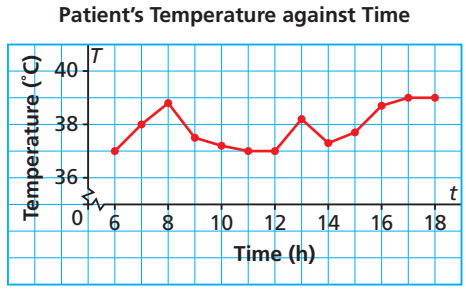
C

21. This table shows the costs to send letters within Canada in 2009.

Mass of Letter	Cost ($)
Up to 30 g	0.54
Greater than 30 g and up to 50 g	0.98
Greater than 50 g and up to 100 g	1.18
Greater than 100 g and up to 200 g	1.96
Greater than 200 g and up to 500 g	2.75

a) Graph the cost of sending a letter as a function of its mass. Did you connect the points? Explain.
b) Write the domain and range of this function.

22. A hospital patient has his temperature taken every hour.



Should the points have been connected? Give reasons for your answer.

23. Is this statement true?
A measure of time can be any real number, so any graph with time as its independent variable should have its points connected.
Explain your answer with examples.

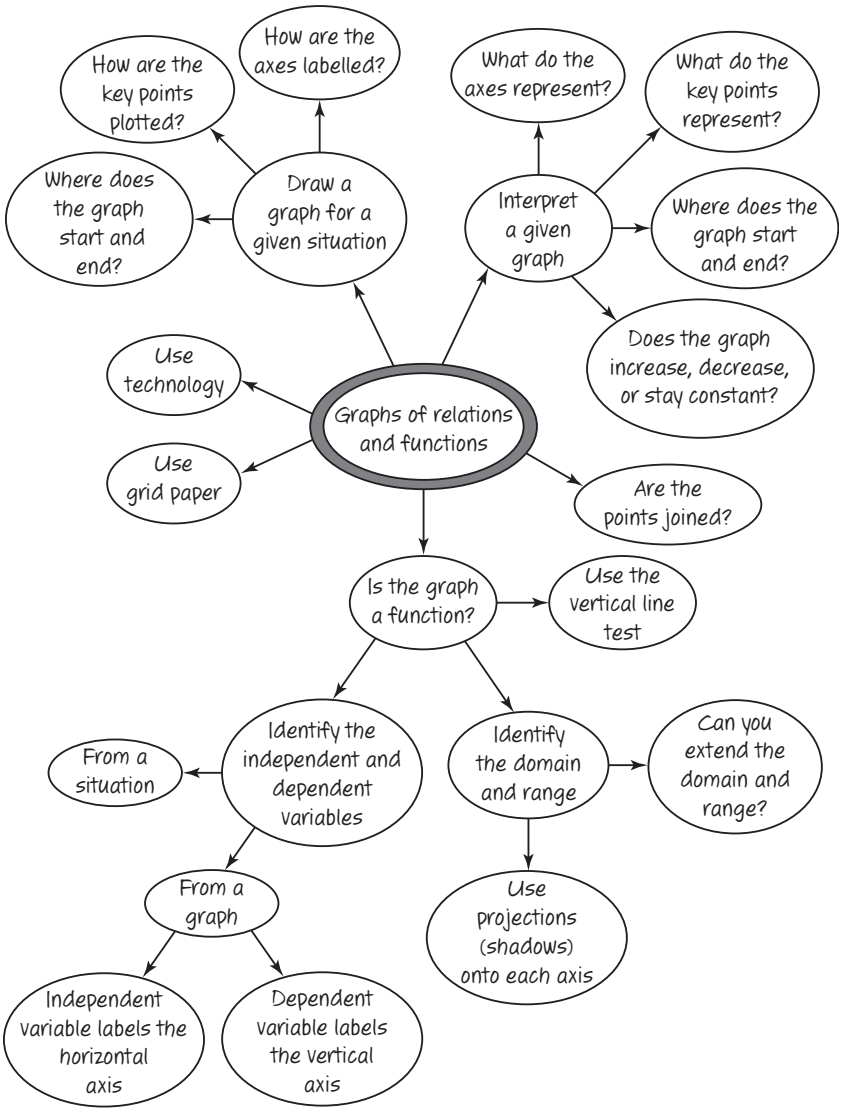
24. Payment scheme 1: A person receives 1¢ on day 1, then each day the payment is doubled.
Payment Scheme 2: A person receives $10 each day.
For both payments, the total money received is a function of the number of days.
a) Make a table of values for each payment scheme.
b) Graph the data.
c) Which payment scheme would you choose if you were receiving the money for 30 days? Explain.

Reflect

Generalize and explain rules for determining whether a graph represents a function. How do you determine the domain and range of a function from its graph? Include examples in your explanation.

CHECKPOINT 2

Connections



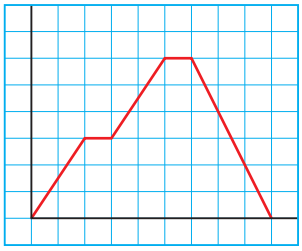
Concept Development

- **In Lesson 5.3**
  - You applied what you know about functions to interpret graphs that represent different situations.
  - You applied what you know about functions to sketch graphs that represent different situations.
- **In Lesson 5.4**
  - You generated data for a relation, then graphed and analyzed the data.
- **In Lesson 5.5**
  - You used the vertical line test on graphs to identify functions.
  - You identified the independent and dependent variables of a function.
  - You graphed tables of values for functions and identified their domains and ranges.
  - You connected points on a graph if all real-number values of the variables were permitted.

Assess Your Understanding

5.3

1. Copy the graph below. Choose labels for each axis, then describe a situation the graph could represent. Justify your description.



5.4

2. **a)** Use technology or grid paper to graph these data for people up to the age of 18.

b) Should you join the points? Explain your reasoning.

c) What are the domain and range of these data?

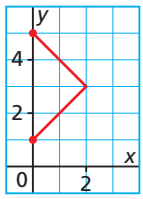
d) Suppose data for more people, up to the age of 18, with different masses were graphed. Would there be any restrictions on the domain and range? If your answer is yes, state the restrictions. If your answer is no, explain why no restrictions exist.

Age (years)	Mass (kg)
14	45
14	50
15	56
15	64
17	65
18	90

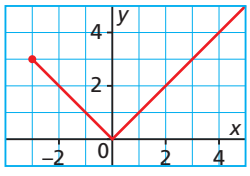
5.5

3. Which graphs represent functions? Justify your answer. Write the domain and range of each graph.

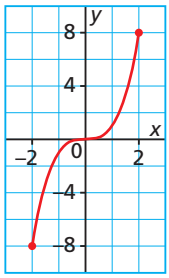
a)



b)



c)



STUDY GUIDE

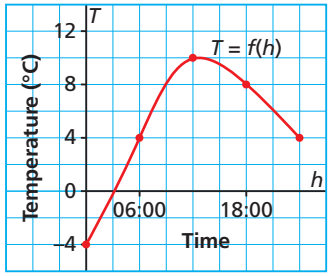
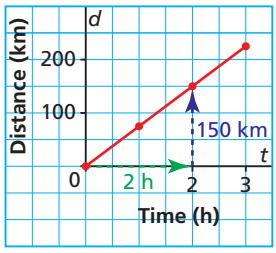
CONCEPT SUMMARY

Big Ideas	Applying the Big Ideas
■ A relation associates the elements of one set with the elements of another set.	This means that: ■ A relation may be represented as: a rule, a table, a set of ordered pairs, an arrow diagram, and a graph. The set of first elements is the domain and the set of related second elements is the range.
■ A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.	■ For a function, each element of the domain is associated with exactly one element of the range.
■ A linear function has a constant rate of change and its graph is a non-vertical straight line.	■ For a linear function, a constant change in the independent variable results in a constant change in the dependent variable, and any vertical line drawn through the graph intersects the graph at no more than one point.

Reflect on the Chapter

- What is a relation? What is a function? Create a graphic organizer to show their common characteristics, and those that are unique.
- How can the properties of linear functions be used to solve real-world problems? Include examples with your explanation.

SKILLS SUMMARY

Skill	Description	Example
Determine the domain and range of a function. [5.2, 5.4, 5.5, 5.7]	The domain is the set of first elements of the ordered pairs. The range is the set of second elements. For a graph, the domain is the set of values of the independent variable. The range is the set of values of the dependent variable.	$\{(-1, 3), (0, 5), (1, 7), (2, 9), (3, 11)\}$ For this set of ordered pairs, the domain is: $\{-1, 0, 1, 2, 3\}$; the range is: $\{3, 5, 7, 9, 11\}$ For the graph below: The domain is all possible times in one day. The range is: $-4 \leq T \leq 10$ 
Determine the rate of change of the graph of a linear function. [5.6, 5.7]	The rate of change is: $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$ The rate of change is positive when the graph goes up to the right. The rate of change is negative when the graph goes down to the right.	Distance against Time  The rate of change is: $\frac{150 \text{ km}}{2 \text{ h}} = 75 \text{ km/h}$
Determine the intercepts of the graph of a linear function. [5.7]	The x-intercept is the value of x when y or $f(x)$ is 0. The y-intercept is the value of y when x is 0.	For the linear function $f(x) = -2x + 5$, When $f(x) = 0$: $0 = -2x + 5$ $2x = 5$ $x = 2.5$ The x-intercept is 2.5. When $x = 0$: $f(0) = -2(0) + 5$ $f(0) = 5$ The y-intercept is 5.

REVIEW

5.1

1. This table shows some Northwest Coast artists and their cultural heritage.

Artist	Heritage
Bob Dempsey	Tlingit
Dorothy Grant	Haida
Bill Helin	Tsimshian
John Joseph	Squamish
Judith P. Morgan	Gitxsan
Bill Reid	Haida
Susan Point	Salish

a) Describe the relation in words.
b) Represent this relation:
i) as a set of ordered pairs
ii) as an arrow diagram

2. Here is a list of some chemical elements and their atomic numbers:
hydrogen (1), oxygen (8), iron (26), chlorine (17), carbon (6), silver (47)
For each association below, use these data to represent a relation in different ways.
a) has an atomic number of
b) is the atomic number of

5.2

3. Which sets of ordered pairs represent functions? What strategies did you use to find out?
a) $\{(4, 3), (4, 2), (4, 1), (4, 0)\}$
b) $\{(2, 4), (-2, 4), (3, 9), (-3, 9)\}$
c) $\{(2, 8), (3, 12), (4, 16), (5, 20)\}$
d) $\{(5, 5), (5, -5), (-5, 5), (-5, -5)\}$

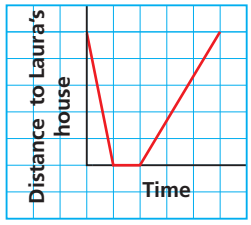
4. Write in function notation.
a) $y = -4x + 9$ **b)** $C = 12n + 75$
c) $D = -20t + 150$ **d)** $P = 4s$

5. The function $P(n) = 5n - 300$ describes the profit, P dollars, for a school dance when n students attend.
a) Write the function as an equation in 2 variables.
b) Identify the independent variable and the dependent variable. Justify your choices.
c) Determine the value of $P(150)$. What does this number represent?
d) Determine the value of n when $P(n) = 700$. What does this number represent?

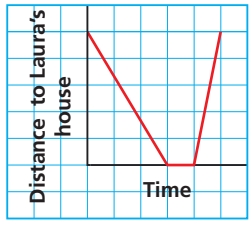
5.3

6. **a)** Laura cycles home from school, then walks back to school. Which graph best matches this situation? Explain your choice.

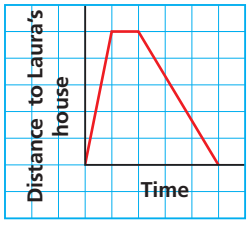
Graph A



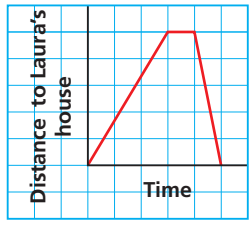
Graph B



Graph C

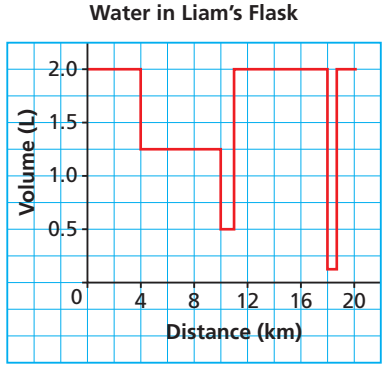


Graph D



b) Choose one of the graphs in part a that did not describe Laura's journey. Describe a possible situation for the graph.

7. This graph shows the volume of water in Liam's flask as he hikes the Trans Canada trail.



a) Describe what is happening for each line segment of the graph.
b) How many times did Liam fill his flask?

c) How much water was in Liam's flask at the start of his hike?
d) Identify the dependent and independent variables.

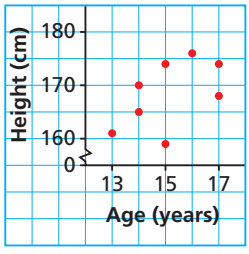
5.4

8. The data below show how the temperature of boiling water as it cools is related to time.
a) Graph the data. Did you join the points? Why or why not?
b) Does the graph represent a function? How can you tell?

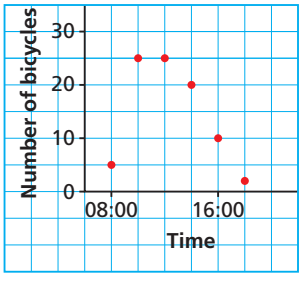
Time (min)	Temperature (°C)
0	89
5	78
10	69
15	62
20	57
25	53
30	50

5.5

9. Which of these graphs represents a function? Justify your answer.
Write the domain and range for each graph.
a) **Heights and Ages of 8 Students**

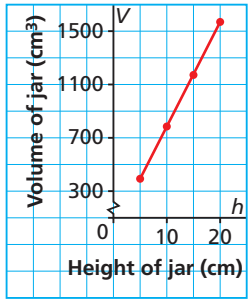


b) **Number of Bicycles at School**

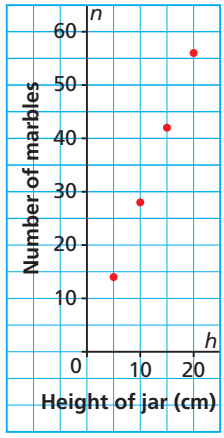


10. For the graphs below:
a) What does each graph represent?
b) Identify the independent and dependent variables.
c) Write the domain and range for each graph. Estimate when necessary. Are there any restrictions on the domain and range? Explain.
d) Why are the points joined on one graph but not on the other?

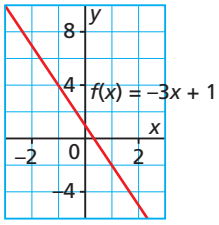
i) **Graph A**



ii) **Graph B**



11. This is a graph of the function $f(x) = -3x + 1$.



a) Determine the range value when the domain value is 1.
b) Determine the domain value when the range value is 4.

12. Sketch a graph of a function that has each domain and range.
a) domain: $-1 \leq x \leq 5$; range: $0 \leq y \leq 3$
b) domain: $x \leq 1$; range: $-2 \leq y \leq 2$

5.6

13. Which sets of ordered pairs represent linear relations? Explain your answers.
a) $\{(1, 5), (5, 5), (9, 5), (13, 5)\}$
b) $\{(1, 2), (1, 4), (1, 6), (1, 8)\}$
c) $\{(-2, -3), (-1, -2), (2, 1), (4, -3)\}$

14. **a)** For each equation, create a table of values when necessary, then graph the relation.

i) $x = 3$

ii) $y = 2x^2 + 3$

iii) $y = 2x + 3$

iv) $y = 3$

v) $y = 3x$

vi) $x + y = 3$

b) Which equations in part a represent linear relations? How do you know?

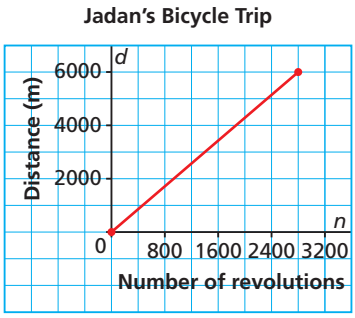
15. Isabelle manages her diabetes by taking insulin to control her blood sugar. The number of units of insulin taken, N, is given by the equation $N = \frac{1}{15}g$, where g represents the number of grams of carbohydrates consumed.

a) Explain why the equation represents a linear relation.

b) State the rate of change. What does it represent?

5.7

16. This graph shows the distance, d metres, travelled by Jadan on her bicycle as a function of the number of wheel revolutions, n, as she rode from Whitehorse to the Grey Mountain Road lookout in the Yukon.



a) How far was Jadan from the lookout when she started her bicycle trip?

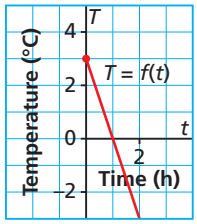
b) Write the domain and range.

c) Determine the rate of change. What does it represent?

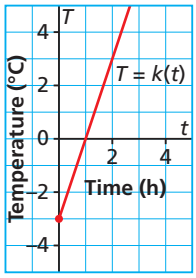
d) Use your answer to part c to determine the diameter of a bicycle wheel.

17. These graphs show the temperature, T degrees Celsius, as a function of time, t hours. Match each graph with its vertical intercept and rate of change.

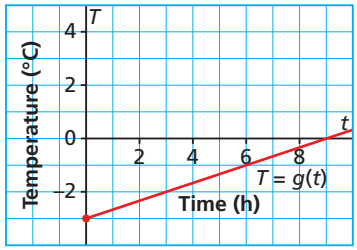
a)



b)



c)

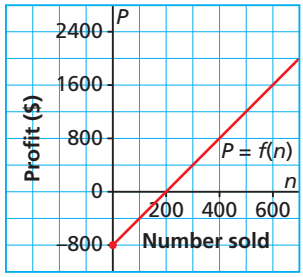


i) $-3°C; \frac{1}{3}°C/h$

ii) $3°C; -3°C/h$

iii) $-3°C; 3°C/h$

18. This graph shows the profit, P dollars, on a company's sale of n baseball caps.



a) How many baseball caps have to be sold before the company begins to make a profit?

b) What is the profit on the sale of each baseball cap?

c) How many caps have to be sold to make each profit?

i) \$600 **ii)** \$1200

d) In part c, when the profit doubles why does the number of baseballs caps sold not double?

PRACTICE TEST

For questions 1 and 2, choose the correct answer: A, B, C, or D

1. For the function $f(x) = 3 - 6x$, what is the value of $f(-3)$?

A. 1 **B.** 21 **C.** -15 **D.** 0

2. Which equation does *not* represent a linear function?

A. $f(x) = 5$ **B.** $f(x) = 5x$ **C.** $f(x) = 5x^2$ **D.** $f(x) = -5$

3. For each relation represented below:

i) State whether it is a function and how you know.

ii) If the relation is a function:
State its domain and range.
Represent the function in a different way.
State whether it is a linear function and how you know.

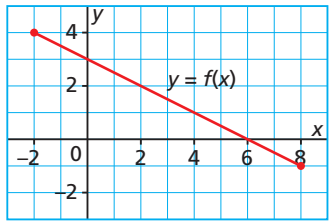
iii) If the relation is a linear function:
Identify the dependent and independent variables.
Determine the rate of change.

a) $\{(2, 5), (-3, 6), (1, 5), (-1, 4), (0, 2)\}$

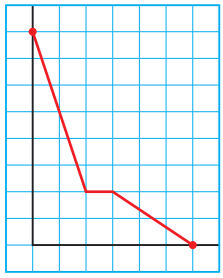
b)

n	s
2	4
-1	1
1	1
-3	9

c)



4. Describe a possible situation for this graph.
Label the axes and give the graph a title.
Justify your description.



5. This table of values shows how the time to cook a turkey is related to its mass.

Mass (kg)	Time (h)
4	2.5
6	3.0
8	3.5
10	4.0

a) Why is this relation a function?

b) Identify the dependent and the independent variables. Justify your choice.

c) Graph the data. Did you connect the points? Explain.

d) Determine the domain and range of the graph. Could you extend the graph? Identify and explain any restrictions on the domain and range. Explain.

e) Determine the rate of change for this function. What does it represent?

f) For how long should you cook a turkey with mass 7 kg?