

Order of Operations with Fractions

Developing: Evaluate each expression.

$$1) \frac{1}{2} + 1\frac{3}{5} \div 2\frac{5}{6} = \frac{1}{2} + \frac{8}{5} \times \frac{6}{17}$$

$$= \frac{1 \times 85}{2 \times 85} + \frac{48 \times 2}{85 \times 2}$$

$$= \frac{85}{170} + \frac{96}{170}$$

$$= \frac{181}{170} = \boxed{1\frac{11}{170}}$$

$$2) \frac{-1}{2} \div \left(-2\frac{3}{4} + 2\frac{1}{4} \right) = \frac{-1}{2} \div \left(\frac{-11}{4} + \frac{9}{4} \right)$$

$$= \frac{-1}{2} \div \frac{-2}{4}$$

$$= \frac{-1}{2} \div \frac{-1}{2}$$

$$= \boxed{1}$$

$$3) \frac{-1}{3} \times 2\frac{2}{3} \times 2 = \frac{-1}{3} \times \frac{8}{3} \times \frac{2}{1}$$

$$= \frac{-16}{9}$$

$$= \boxed{-1\frac{7}{9}}$$

$$4) \frac{2}{3} \times -1\frac{1}{3} + 1 = \frac{2}{3} \times \frac{-4}{3} + \frac{1}{1}$$

$$= \frac{-8}{9} + \frac{1 \times 9}{1 \times 9}$$

$$= \frac{-8}{9} + \frac{9}{9}$$

$$= \boxed{\frac{1}{9}}$$

$$5) \frac{1}{2} \times \frac{-9}{5} \times -2 = \frac{1}{2} \times \frac{-9}{5} \times \frac{-2}{1}$$

$$= \frac{9}{5}$$

$$= \boxed{1\frac{4}{5}}$$

$$6) \left(2\frac{1}{3} - 1\frac{1}{4} \right) \times -3 = \left(\frac{7}{3} - \frac{5}{4} \right) \times \frac{-3}{1}$$

$$= \left(\frac{28}{12} - \frac{15}{12} \right) \times \frac{-3}{1}$$

$$= \frac{13}{4} \times \frac{-3}{1}$$

$$= \frac{-13}{4} = \boxed{-3\frac{1}{4}}$$

$$\begin{aligned}
 7) \left(1\frac{1}{2}\right)^2 - 3\frac{2}{3} &= \frac{3}{2} \times \frac{3}{2} + \frac{11}{3} \\
 &= \frac{9 \times 3}{4 \times 3} + \frac{11 \times 4}{3 \times 4} \\
 &= \frac{27}{12} + \frac{44}{12} \\
 &= \frac{71}{12} = \boxed{5\frac{11}{12}}
 \end{aligned}$$

$$\begin{aligned}
 8) -1\frac{3}{4} + \frac{1}{6} + \frac{5}{4} &= \frac{-7 \times 3}{4 \times 3} + \frac{1 \times 2}{6 \times 2} + \frac{5 \times 3}{4 \times 3} \\
 &= \frac{-21}{12} + \frac{2}{12} + \frac{15}{12} \\
 &= \frac{-4}{12} = \boxed{-\frac{1}{3}}
 \end{aligned}$$

$$\begin{aligned}
 9) 2\frac{5}{6} \div \frac{-1}{3} - 1\frac{3}{4} &= \frac{17}{6} \div \frac{-1}{3} - \frac{7}{4} \\
 &= \frac{17}{24} \times \frac{-3^1}{1} - \frac{7}{4} \\
 &= \frac{-17 \times 2}{2 \times 2} - \frac{7}{4} \\
 &= \frac{-34}{4} - \frac{7}{4} = \frac{-41}{4} = \boxed{-10\frac{1}{4}}
 \end{aligned}$$

$$\begin{aligned}
 10) -1\frac{1}{2} \div \left(-1\frac{3}{4} \times 3\frac{1}{5}\right) &= \frac{-3}{2} \div \left(\frac{-7}{4} \times \frac{16}{5}\right) \\
 &= \frac{-3}{2} \div \frac{-28}{5} \\
 &= \frac{-3}{2} \times \frac{5}{28} \\
 &= \boxed{\frac{15}{56}}
 \end{aligned}$$

Proficient: Evaluate each expression.

$$\begin{aligned}
 11) -2 + \left(\frac{-2}{3} - 1\frac{2}{3}\right) \div \frac{5}{4} &= \frac{-2}{1} + \left(\frac{-2}{3} - \frac{5}{3}\right) \div \frac{5}{4} \\
 &= \frac{-2}{1} + \frac{-7}{3} \times \frac{4}{5} \\
 &= \frac{-2 \times 15}{1 \times 15} + \frac{-28}{15} \\
 &= \frac{-30}{15} + \frac{-28}{15} \\
 &= \frac{-58}{15} = \boxed{-3\frac{13}{15}}
 \end{aligned}$$

$$\begin{aligned}
 12) \frac{-1}{2} \left(-3\frac{2}{5} \times -2 - \frac{4}{5}\right) &= \frac{-1}{2} \left(\frac{17}{5} \times \frac{-2}{1} - \frac{4}{5}\right) \\
 &= \frac{-1}{2} \left(\frac{34}{5} - \frac{4}{5}\right) \\
 &= \frac{-1}{2} \left(\frac{30}{5}\right) \\
 &= \frac{-1}{2} \left(\frac{6}{1}\right) \\
 &= \boxed{-3}
 \end{aligned}$$

$$\begin{aligned}
 13) \left(-1 - 3\frac{1}{2}\right) \times \frac{-1}{3} \times \frac{1}{3} &= \left(\frac{-1}{1} - \frac{7}{2}\right) \times \frac{-1}{3} \times \frac{1}{3} \\
 &= \left(\frac{-2}{2} - \frac{7}{2}\right) \times \frac{-1}{3} \times \frac{1}{3} \\
 &= \frac{-9}{2} \times \frac{-1}{3} \times \frac{1}{3} \\
 &= \boxed{\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 14) \left(\frac{3}{4} \left(-2\frac{1}{3} + 1\frac{3}{5}\right)\right) \div \frac{-1}{2} &= \left(\frac{3}{4} \left(\frac{-7}{3} + \frac{8}{5}\right)\right) \div \frac{-1}{2} \\
 &= \left(\frac{3}{4} \left(\frac{-35}{15} + \frac{24}{15}\right)\right) \div \frac{-1}{2} \\
 &= \frac{1}{2} \left(\frac{-11}{4}\right) \times \frac{-2}{1} \\
 &= \frac{11}{10} = \boxed{1\frac{1}{10}}
 \end{aligned}$$

$$\begin{aligned}
 15) \left(2\frac{1}{3} - 1\frac{1}{2}\right)^2 - \frac{1}{2} &= \left(\frac{7}{3} - \frac{3}{2}\right)^2 - \frac{1}{2} \\
 &= \left(\frac{14}{6} - \frac{9}{6}\right)^2 - \frac{1}{2} \\
 &= \left(\frac{5}{6}\right)^2 - \frac{1}{2} \\
 &= \frac{5}{6} \times \frac{5}{6} - \frac{1 \times 18}{2 \times 18} \\
 &= \frac{25}{36} - \frac{18}{36} = \boxed{\frac{7}{36}}
 \end{aligned}$$

$$\begin{aligned}
 16) 4\frac{1}{2} - \left(3\frac{1}{5} + \frac{1}{2} - 3\right) &= \frac{9}{2} - \left(\frac{16}{5} + \frac{1}{2} - \frac{3}{1}\right) \\
 &= \frac{9}{2} - \left(\frac{32}{10} + \frac{5}{10} - \frac{30}{10}\right) \\
 &= \frac{9 \times 5}{2 \times 5} - \frac{7}{10} \\
 &= \frac{45}{10} - \frac{7}{10} \\
 &= \frac{38}{10} = 3\frac{8}{10} = \boxed{3\frac{4}{5}}
 \end{aligned}$$

$$\begin{aligned}
 17) \left(\frac{-11}{6} - \frac{3}{2} \times -3\frac{1}{2}\right) \div \frac{-3}{2} \\
 \left(\frac{-11}{6} - \frac{3 \times -7}{2}\right) \div \frac{-3}{2} \\
 \left(\frac{-11 \times 2}{6 \times 2} + \frac{21 \times 3}{4 \times 3}\right) \div \frac{-3}{2} \\
 \left(\frac{-22}{12} + \frac{63}{12}\right) \div \frac{-3}{2} \\
 \frac{41}{12} \times \frac{2}{3} = \frac{-41}{18} = \boxed{-2\frac{5}{18}}
 \end{aligned}$$

$$\begin{aligned}
 18) \frac{1}{3} \div \left(\frac{-1}{3} \left(\frac{-1}{4} - -2\frac{3}{5}\right)\right) \\
 = \frac{1}{3} \div \left(-\frac{1}{3} \left(\frac{-1}{4} + \frac{13}{5}\right)\right) \\
 = \frac{1}{3} \div \left(-\frac{1}{3} \left(\frac{-5}{20} + \frac{52}{20}\right)\right) \\
 = \frac{1}{3} \div \left(-\frac{1}{3} \times \frac{47}{20}\right) \\
 = \frac{1}{3} \div \frac{-47}{60} = \frac{1}{3} \times \frac{-20}{47} = \boxed{\frac{-20}{47}}
 \end{aligned}$$

$$\begin{aligned}
 19) 2\frac{1}{3} + 1\frac{2}{3} \div (-2)^3 &= \frac{7}{3} + \frac{5}{3} \div (-8) \\
 &= \frac{7}{3} + \frac{5}{3} \times \frac{-1}{8} \\
 &= \frac{7 \times 8}{3 \times 8} - \frac{5}{24} \\
 &= \frac{56}{24} - \frac{5}{24} = \frac{51}{24} = \frac{17}{8} = \boxed{2\frac{1}{8}}
 \end{aligned}$$

$$\begin{aligned}
 20) \left(2\frac{1}{6} - 3\frac{1}{2}\right) \div \left(2 - -1\frac{1}{5}\right) &= \left(\frac{13}{6} - \frac{7}{2}\right) \div \left(\frac{2}{1} + \frac{1}{5}\right) \\
 &= \left(\frac{13}{6} - \frac{21}{6}\right) \div \left(\frac{10}{5} + \frac{1}{5}\right) \\
 &= \frac{-8}{6} \div \frac{16}{5} \\
 &= \frac{-8}{6} \times \frac{5}{16} \\
 &= \boxed{\frac{-5}{12}}
 \end{aligned}$$

Extending: Evaluate each expression.

$$\begin{aligned}
 21) 3\frac{2}{3} \times 2 \times -2\frac{1}{5} \times \frac{-2}{3} \times \frac{3}{4} \\
 \frac{11}{3} \times \frac{2}{1} \times \frac{-11}{5} \times \frac{-2}{3} \times \frac{3}{4} \\
 = \frac{121}{15} \\
 = \boxed{8\frac{1}{15}}
 \end{aligned}$$

$$\begin{aligned}
 22) -1\frac{1}{6} \times \frac{3}{2} - -2 + 2\frac{3}{5} \times \frac{1}{2} &= \frac{-7}{6} \times \frac{3}{2} + 2 + \frac{13}{5} \times \frac{1}{2} \\
 &= \frac{-7 \times 3}{6 \times 2} + \frac{2 \times 20}{1 \times 20} + \frac{13 \times 1}{5 \times 2} \\
 &= \frac{-35}{20} + \frac{40}{20} + \frac{26}{20} \\
 &= \frac{5}{20} + \frac{26}{20} = \frac{31}{20} = \boxed{1\frac{11}{20}}
 \end{aligned}$$

$$23) \left(1\frac{1}{4} - 3 \div \left(5 - \frac{1}{3}\right)\right) \times \frac{-1}{2} = \left(\frac{5}{4} - 3 \div \left(\frac{5-1}{3}\right)\right) \times \frac{-1}{2}$$

$$= \left(\frac{5}{4} - 3 \div \left(\frac{15-1}{3}\right)\right) \times \frac{-1}{2}$$

$$= \left(\frac{5}{4} - 3 \times \frac{3}{14}\right) \times \frac{-1}{2}$$

$$= \left(\frac{5}{4} - \frac{9}{14}\right) \times \frac{-1}{2} = \left(\frac{70-36}{56}\right) \times \frac{-1}{2}$$

$$= \frac{1734}{56} \times \frac{-1}{2} = \boxed{\frac{-17}{56}}$$

$$24) \left(\frac{-3}{2} + \left(\frac{1}{3}\right)^3\right) \div \left(\frac{1}{3} - \frac{5}{3}\right) = \left(\frac{-3}{2} + \frac{1}{27}\right) \div \left(\frac{-4}{3}\right)$$

$$= \left(\frac{-81}{54} + \frac{2}{54}\right) \times \frac{-3}{4}$$

$$= \frac{-79}{1854} \times \frac{-3}{4}$$

$$= \frac{+79}{72} = \boxed{\frac{7}{72}}$$

$$25) \left(\left(\frac{2}{3}\right)^2 - (-1 - -2)\right) \times \frac{-6}{5}$$

$$= \left(\frac{4}{9} - \frac{1}{1}\right) \times \frac{-6}{5}$$

$$= \left(\frac{4}{9} - \frac{9}{9}\right) \times \frac{-6}{5}$$

$$= \frac{18}{39} \times \frac{-6}{5}$$

$$= \boxed{\frac{2}{3}}$$

$$26) 3\frac{1}{4} \div \frac{-1}{6} - -1\frac{1}{3} - (-1)^3 \times 2\frac{4}{5}$$

$$= \frac{13}{4} \times \frac{-6}{1} + \frac{4}{3} - (-1) \times \frac{14}{5}$$

$$= \frac{-39}{2} + \frac{4}{3} + \frac{1}{1} \times \frac{14}{5}$$

$$= \frac{-39}{2} + \frac{4}{3} + \frac{14}{5}$$

$$= \frac{-585}{30} + \frac{40}{30} + \frac{84}{30} = \frac{-461}{30} = \boxed{-15\frac{11}{30}}$$

$$27) \frac{-1}{4} - -2 + -3\frac{2}{3} \times 1\frac{1}{3} + \frac{3}{2} + 1$$

$$= \frac{-1}{4} + \frac{2}{1} + \frac{-11}{3} \times \frac{4}{3} + \frac{3}{2} + 1$$

$$= \frac{-1}{4} + \frac{2}{1} + \frac{-44}{9} + \frac{3}{2} + 1$$

$$= \frac{-9}{36} + \frac{72}{36} + \frac{-176}{36} + \frac{54}{36} + \frac{36}{36}$$

$$= \boxed{\frac{-23}{36}}$$

$$28) -3\frac{1}{5} \times \frac{1}{2} - \left(\frac{-1}{2} - \left(-2 - 1\frac{1}{2}\right)\right) - \frac{2}{3}$$

$$= \frac{-8}{5} \times \frac{1}{2} - \left(\frac{-1}{2} - \left(\frac{-2}{1} - \frac{3}{2}\right)\right) - \frac{2}{3}$$

$$= \frac{-8}{5} - \left(\frac{-1}{2} - \left(\frac{-4}{2} - \frac{3}{2}\right)\right) - \frac{2}{3}$$

$$= \frac{-8}{5} - \left(\frac{-1}{2} - \frac{-7}{2}\right) - \frac{2}{3} = \frac{-8}{5} - \left(\frac{6}{2}\right) - \frac{2}{3}$$

$$= \frac{-24}{15} - \frac{45}{15} - \frac{10}{15} = \frac{-79}{15} = \boxed{-5\frac{4}{15}}$$

$$29) 2\frac{1}{2} + \left(1 - 1\frac{1}{2}\right)^2 \div \frac{1}{5} \times 3\frac{1}{2}$$

$$= \frac{5}{2} + \left(\frac{1}{1} - \frac{3}{2}\right)^2 \div \frac{1}{5} \times \frac{7}{2}$$

$$= \frac{5}{2} + \left(\frac{2}{2} - \frac{3}{2}\right)^2 \times \frac{5}{1} \times \frac{7}{2}$$

$$= \frac{5}{2} + \left(\frac{-1}{2}\right)^2 \times \frac{5}{1} \times \frac{7}{2} = \frac{5}{2} + \frac{1}{4} \times \frac{5}{1} \times \frac{7}{2}$$

$$= \frac{5}{2} + \frac{35}{8} = \frac{20}{8} + \frac{35}{8} = \frac{55}{8} = \boxed{6\frac{7}{8}}$$

$$30) \left(6 - 2\frac{1}{4}\right) \div \left(\frac{9}{5} \times \frac{-11}{6}\right) \times 2\frac{2}{5} - 2\frac{1}{6}$$

$$= \left(\frac{24}{4} - \frac{1}{4}\right) \div \left(\frac{-33}{10}\right) \times \frac{12}{5} - \frac{13}{6}$$

$$= \frac{18}{4} \times \frac{-105}{33} \times \frac{12}{5} - \frac{13}{6}$$

$$= \frac{-30}{11} - \frac{13}{6} = \frac{-180}{66} - \frac{143}{66}$$

$$= \frac{-323}{66} = \boxed{-4\frac{59}{66}}$$