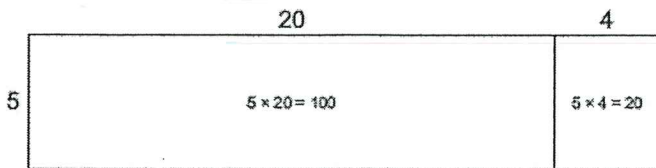


Math 9
Lesson P3: Adding & Subtracting Polynomials

The Distributive Property

We can use this diagram to model 5×24 .



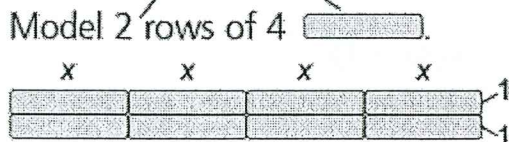
This diagram shows:

$$\begin{aligned}
 5 \times 24 &= 5 \times (20 + 4) \\
 &= (5 \times 20) + (5 \times 4) \\
 &= 100 + 20 \\
 &= 120
 \end{aligned}$$

We multiply the term outside the brackets by each term inside the brackets, then find the sum.

Multiplying a Polynomial by a Constant:

To multiply $2(4x)$ with algebra tiles:



There are 8 x-tiles. So, $2(4x) = 8x$

Example #1: Multiply the following polynomial expressions using different methods.

a) $3(2s - 6)$

3 groups of $2s - 6$



$3(2s - 6) = 6s - 18$



$$\begin{array}{r}
 2s \quad -6 \\
 3 \boxed{6s \quad -18}
 \end{array}$$

$3(2s - 6)$

$= 6s - 18$

b) $-2(x^2 - 4x - 3)$

$= -2x^2 + 8x + 6$


2 groups of $(x^2 - 4x - 3)$, but since the 2 is negative, flip everything in the bracket

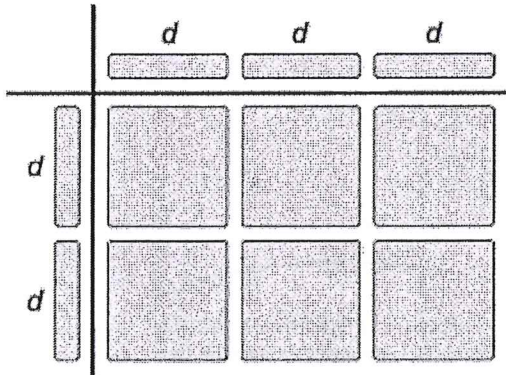


Multiplying a Polynomial by a Monomial:

To multiply $2d(3d)$ with algebra tiles:

Draw 2 adjacent sides of a rectangle.

Position  tiles to show side lengths $2d$ and $3d$.



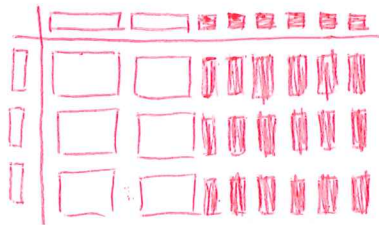
$d \times d = d^2$, so use a d^2 -tile.

Then fill the rectangle with tiles.

We used 6 d^2 -tiles to fill the rectangle. So, $2d(3d) = 6d^2$

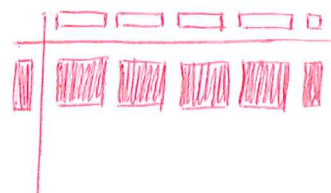
Example #2: Multiply the following polynomial expressions using different methods.

a) $3s(2s - 6)$



$= 6s^2 - 18s$

b) $-x(4x + 1)$



$= -4x^2 - x$

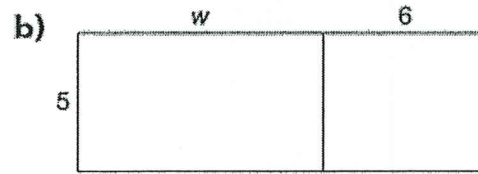
Practice

1. Which multiplication sentence does each rectangle represent?



$$4(m + 4) = (4 \times m) + (4 \times 4)$$

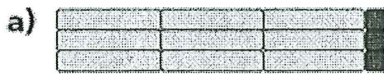
$$= 4m + 16$$



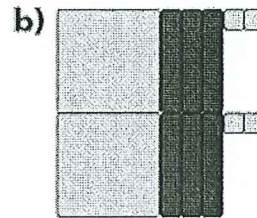
$$5(w + 6) = (5 \times w) + (5 \times 6)$$

$$= 5w + 30$$

2. Write the multiplication sentence modelled by each set of tiles.



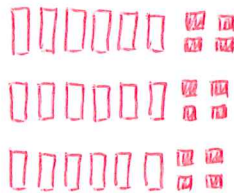
$$3(3x - 1) = 9x - 3$$



$$2(x^2 - 3x + 2) = 2x^2 - 6x + 4$$

3. Sketch algebra tiles to multiply. Write the product each time.

a) $3(6r - 4) = 18r - 12$



b) $2(-2b^2 - b + 3) = -4b^2 - 2b + 6$

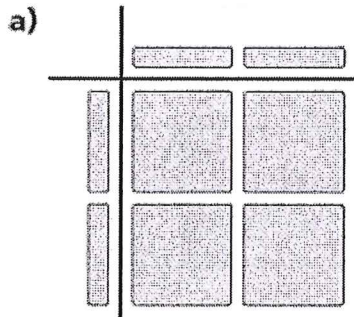


4. Multiply.

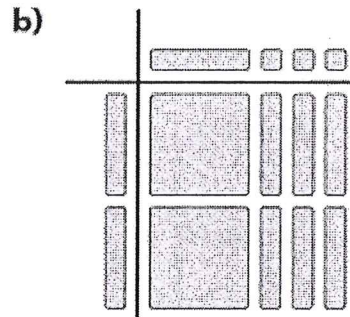
a) $6(-4t^2 + 3)$
 $= 6(-4t^2) + 6(3)$
 $= -24t^2 + 18$

b) $-8(-3k^2 - 2k + 4)$
 $= -8(-3k^2) - 8(-2k) - 8(4)$
 $= 24k^2 + 16k - 32$

1. Write the multiplication sentence modelled by each set of tiles.



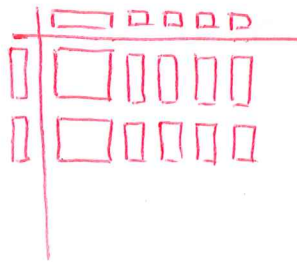
$$(2x)(2x) = 4x^2$$



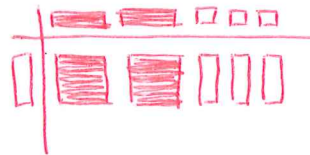
$$2x(x+3) = 2x^2 + 6x$$

2. Sketch algebra tiles to multiply. Write the product each time.

a) $2s(s+4) = 2s^2 + 8s$



b) $t(-2t+3) = -2t^2 + 3t$



3. Multiply.

a) $4r(5r-1)$
 $= (4r)(5r) + (4r)(-1)$
 $= 20r^2 + (-4r)$
 $= 20r^2 - 4r$

b) $7s(-3s+6)$
 $= 7s(-3s) + 7s(6)$
 $= -21s^2 + 42s$

c) $-6t(t-3)$
 $= -6t(t) - 6t(-3)$
 $= -6t^2 + 18t$

d) $-8u(-6u+7)$
 $= -8u(-6u) - 8u(7)$
 $= 48u^2 - 56u$

What Did the Girl Mushroom Say About the Boy Mushroom After Their First Date ?



For each exercise below, multiply the polynomial by the monomial. Find your answer in the set of answers under the exercise and notice the letter next to it. Write this letter in the box that contains the number of that exercise.



- 1 $5(2n^2 + n) = 10n^2 + 5n$
- 2 $3n(8n^2 - 2n) = 24n^3 - 6n^2$
- 3 $n^2(4n - 3) = 4n^3 - 3n^2$
- 4 $-2n(4 + 5n^3) = -8n - 10n^4$
- 5 $-6n^2(4n^2 - 9) = -24n^4 + 54n^2$

Answers:

- B $-24n^4 - 54n$
- T $24n^3 - 4n$
- R $-24n^4 + 54n^2$
- U $4n^3 - 3n^2$
- S $10n^2 + 5n$
- L $24n^3 - 6n^2$
- O $-8n - 6n^3$
- A $-8n - 10n^4$

- 6 $4a(a^2 - 2a + 3) = 4a^3 - 8a^2 + 12a$
- 7 $-2a^2(9 - a - 4a^2) = -18a^2 + 2a^3 + 8a^4$
- 8 $a^2b(a^2 - b^2) = a^4b - a^2b^3$
- 9 $-3ab^2(a^3b^2 - 2a^2b) = -3a^4b^4 + 6a^3b^3$
- 10 $2ab(a^2 + 4ab - 3b^2) = 2a^3b + 8a^2b^2 - 6ab^3$

Answers:

- M $4a^3 - 8a^2 + 10$
- H $-18a^2 + 2a^3 + 8a^4$
- E $2a^3b + 8a^2b^2 - 6ab^3$
- I $2a^3b + 8ab^2 - 4ab$
- A $a^4b - a^2b^3$
- G $4a^3 - 8a^2 + 12a$
- W $-18a^2 + 2a^3 + 6a^5$
- L $-3a^4b^4 + 6a^3b^3$

- 11 $x^2y(2x^2 - 4xy + y^2) = 2x^4y - 4x^3y^2 + x^2y^3$
- 12 $-2xy^2(2x^4 - 5x^2y^2 - 3y^4) = -4x^6y^2 + 10x^3y^4 + 6xy^6$
- 13 $4x^3y(-x^2y + 2xy - 5xy^2) = -4x^5y^2 + 8x^4y^2 - 20x^4y^3$
- 14 $-x^2y^3(7xy^3 - x^2y^2 + 3x^3y) = -7x^5y^6 + x^4y^5 - 3x^5y^4$
- 15 $3x^2y^2(2x^4y^2 - 3x^2y - 1) = 6x^6y^4 - 9x^4y^3 - 3x^2y^2$

Answers:

- N $-4x^5y^2 + 10x^3y^4 + 6xy^6$
- S $2x^4y - 4x^2y^3 + x^2y^4$
- E $-4x^5y^2 + 8x^4y^2 - 20x^4y^3$
- U $-4x^5y^2 + 10x^2y^4 - 20x^2y^3$
- Y $2x^4y - 4x^3y^2 + x^2y^3$
- F $6x^6y^4 - 9x^4y^3 - 3x^2y^2$
- T $-7x^5y^6 + x^5y^4 - 3x^3y^4$
- I $-7x^5y^6 + x^4y^5 - 3x^5y^4$

KEY

| | | | | | | | | | | | | | | |
|---|----|---|---|----|---|---|---|----|---|----|---|----|---|----|
| 7 | 10 | 1 | 5 | 13 | 4 | 9 | 2 | 11 | 8 | 15 | 3 | 12 | 6 | 14 |
| H | E | S | R | E | A | L | L | Y | A | F | U | N | G | I |

Dividing a Polynomial by a Constant:

We can use algebra tiles to divide a polynomial by a constant.

To divide: $(-8x) \div 2$

Arrange 8  into 2 equal rows.



In each row there are 4 .

So, $(-8x) \div 2 = -4x$

Example #3: Divide the following polynomial expressions using different methods.

a) $(6s - 12) \div 3$

b) $\frac{4x^2 - 2x - 8}{2}$

Dividing a Polynomial by a Monomial:

To divide a polynomial by a monomial, we use what we already know about division.

To divide: $\frac{6a^2}{3a}$

We write the fraction as a product of 2 fractions, then simplify each fraction.

$$\begin{aligned}\frac{6a^2}{3a} &= \frac{6}{3} \times \frac{a^2}{a} \\ &= 2 \times \frac{a \times a^1}{a^1} \\ &= 2 \times a \\ &= 2a\end{aligned}$$

a is a common factor of the numerator and the denominator.

$2a$ is the quotient of $\frac{6a^2}{3a}$.

Example #4: Divide the following polynomial expressions.

a) $(12a^2 + 8a) \div 2a$

b) $\frac{-15b^2 + 20b}{-5b}$

Practice

5. Which of these quotients is modelled by the tiles below?



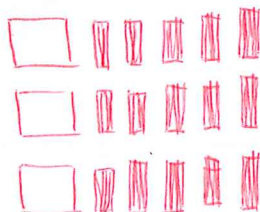
a) $(15x - 9) \div 3$

b) $(-15x - 9) \div 3$

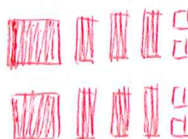
c) $(-15x + 9) \div 3$

6. Sketch algebra tiles to divide. Write the quotient each time.

a) $(3h^2 - 15h) \div 3 = h^2 - 5h$



b) $(-2a^2 - 6a + 4) \div 2 = -a^2 - 3a + 2$



7. Divide.

a) $\frac{-10z^2 + 15}{5} = -2z^2 + 3$

b) $\frac{7x^2 - 7x + 21}{-7} = -x^2 + x - 3$

4. Divide.

a) $\frac{12v^2}{4v} = 3v$

b) $\frac{15w^2}{-3w} = -5w$

c) $\frac{-28x^2}{-7x} = 4x$

5. Divide.

a) $\frac{18y^2 + 12y}{2y} = 9y + 6$

b) $\frac{-32z^2 + 24z}{-8z} = 4z - 3$

c) $\frac{15n^2 + 21n}{-3n} = -5n - 7$

What Did the Carpenters Call Their Bass Quartet?

Simplify each expression. Assume that no divisor equals zero. Find your answer in the set of answers under the exercise and cross out the box above it. When you finish, the answer to the title question will remain.

| | | |
|------------------------------------|--|---|
| ① $\frac{6x+9}{3} = 2x+3$ | ⑥ $\frac{12v^5-27v^4}{3v^2} = 4v^3-9v^2$ | ⑪ $\frac{8a^3+4a^2-24a}{4a} = 2a^2+a-b$ |
| ② $\frac{18x^2-50}{2} = 9x^2-25$ | ⑦ $\frac{30u^4-6u}{-6u} = -5u^3+1$ | ⑫ $\frac{21ab^3+14a^2b+35a^4}{7a} = 3b^3+2ab+5a^3$ |
| ③ $\frac{12x^2+20x}{4x} = 3x+5$ | ⑧ $\frac{u^2v+uv^2}{uv} = u+v$ | ⑬ $\frac{2a^3b-6a^2b^2+16ab^3}{-2ab} = -a^2+3ab-8b^2$ |
| ④ $\frac{20x^3+5x^2}{5x} = 4x^2+x$ | ⑨ $\frac{8uv^4-14u^2v^3}{2uv} = 4v^3-7uv^2$ | ⑭ $\frac{45a^2b^4-60a^3b^2-15a^2b}{15a^2b} = 3b^3-4ab-1$ |
| ⑤ $\frac{2x^3-7x^2}{x^2} = 2x-7$ | ⑩ $\frac{-10u^3v^2+5u^2v^5}{-5u^2v} = 2uv-v^4$ | ⑮ $\frac{15a^5b^4+3a^4b^5-6a^3b^6}{3a^2b^3} = 5a^3b+a^2b^2-2ab^3$ |
| A 4 | E 8 | U |
| T 5 | B 9 | R |
| O 3 | 7 | O |
| S 25 | 4 | A |
| H 25 | 3 | E |
| 2 | 25 | B |
| 9x ² -25 | 4v ³ -7uv ² | 2a ² -a-1 |
| 3x-25 | 2uv-5v | 3b ³ -4ab-1 |
| 4x ² +x | 4v ³ -9v ² | 3b ³ +2ab+5a ³ |
| 2x+3 | u+v | -a ² -4ab+1 |
| 4x ² +5 | 4v ³ +2v | 2a ² +a-6 |
| 3x+5 | 2uv-v ⁴ | 3b ³ -3ab-8b ² |
| 2x-7 | -5u ³ +1 | 5a ³ b+a ² b ² -2ab ³ |
| | | 5a ³ b+a ² b ² -4ab ² |
| | | -a ² +3ab-8b ² |

The tuba 4 \Rightarrow "The two by four"