

Math 9
Lesson P2: Adding & Subtracting Polynomials

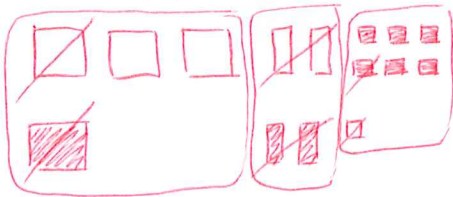
There are several different methods that can be used to add or subtract polynomials:

- Algebra tiles
- Symbolically/Horizontally
- Vertically

Example #1: Add the following polynomials using different methods.

$$(3s^2 + 2s - 6) + (-s^2 - 2s + 1)$$

- Algebra Tiles



$$\square \square \square \quad \square \square \square \quad = \boxed{2s^2 - 5}$$

- Symbolically/Horizontally

$$\cancel{3s^2} + 2s - 6 - \cancel{s^2} - 2s + 1$$

$$3s^2 - s^2 + 2s - 2s - 6 + 1$$

$$\begin{array}{r} \checkmark \qquad \qquad \checkmark \qquad \qquad \checkmark \\ 3s^2 - s^2 + 2s - 2s - 6 + 1 \\ \hline 2s^2 + 0s - 5 = \boxed{2s^2 - 5} \end{array}$$

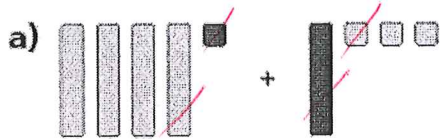
- Vertically

$$\begin{array}{r} 3s^2 + 2s - 6 \\ + \quad -s^2 - 2s + 1 \\ \hline 2s^2 + 0s - 5 = \boxed{2s^2 - 5} \end{array}$$

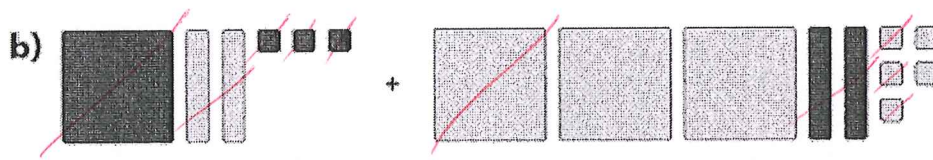
Practice

1. Write the addition sentence modelled by each set of tiles.

Use the variable x .



$$(4x - 1) + (-x + 3) = 3x + 2$$

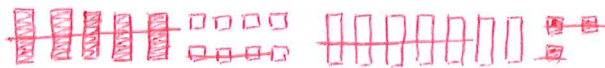


$$(-x^2 + 2x - 3) + (3x^2 - 2x + 5) = 2x^2 + 2$$

2. Sketch algebra tiles to model each sum.

Then write the sum.

a) $(-5w + 8) + (7w - 3) = \underline{2w + 5}$



Remaining tiles: 2 vertical w-tiles, 5 small square tiles

b) $(-6t^2 - 3t + 2) + (4t^2 - t + 1) = \underline{-2t^2 - 4t + 3}$



Remaining tiles: 2 large square tiles, 4 vertical t-tiles, 3 small square tiles

3. Add horizontally.

$$\begin{aligned} \text{a) } & (2r - 3) + (3r - 1) \\ & = 2r - 3 + 3r - 1 \\ & = 2r + 3r - 3 - 1 \\ & = \underline{5r - 4} \end{aligned}$$

$$\begin{aligned} \text{b) } & (7h^2 - 2h) + (-4h^2 + 9h - 4) \\ & = \underline{7h^2 - 2h - 4h^2 + 9h - 4} \\ & = \underline{7h^2 - 4h^2 - 2h + 9h - 4} \\ & = \underline{3h^2 + 7h - 4} \end{aligned}$$

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

$$2 + 3 = \underline{5} \text{ and } -3 + (-1) = \underline{-4}$$

$$\begin{aligned} \text{c) } & (-2y^2 + 6y - 1) + (2y^2 - 6y + 5) \\ & = \underline{-2y^2 + 6y - 1 + 2y^2 - 6y + 5} \\ & = \underline{-2y^2 + 2y^2 + 6y - 6y - 1 + 5} \\ & = \underline{4} \end{aligned}$$

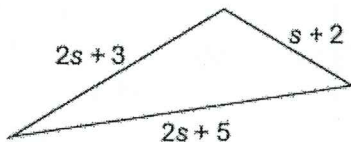
4. Add vertically.

$$\begin{array}{r} \text{a) } (9r + 7) + (2r - 3) \\ 9r + 7 \\ + 2r - 3 \\ \hline \underline{11r + 4} \end{array}$$

$$\begin{array}{r} \text{b) } (-a^2 + 4a) + (-3a^2 + 2a - 5) \\ -a^2 + 4a \\ + -3a^2 + 2a - 5 \\ \hline \underline{-4a^2 + 6a - 5} \end{array}$$

$$\begin{array}{r} \text{c) } (8v - 2v^2 - 3) + (9 + 6v^2 - 10v) \\ -2v^2 + 8v - 3 \\ + 6v^2 - 10v + 9 \\ \hline \underline{4v^2 - 2v + 6} \end{array}$$

5. Find the perimeter of this triangle.



$$\begin{aligned} \text{Perimeter} &= \underline{(2s+3) + (s+2) + (2s+5)} \\ &= \underline{2s+3+s+2+2s+5} \\ &= \underline{2s+s+2s+3+2+5} \\ &= \underline{5s+10} \end{aligned}$$

Perimeter is the distance around a shape. To find the perimeter, add the side lengths.

Remove the brackets.

Group like terms.

Add coefficients of like terms.

Why is an Idea Like the Pacific?

For each exercise below, add the polynomials. Find your answer at the bottom of the page and write the letter of that exercise above it.

Ⓓ $6x + 9$

$$\frac{x-1}{7x+8}$$

Ⓘ $3x - 4$

$$\frac{5x-7}{8x-11}$$

ⓐ $8x^2 + 2x + 1$

$$\frac{x^2 - 4x + 7}{9x^2 - 2x + 8}$$

Ⓢ $-5x^2 - 5x + 3$

$$\frac{6x^2 - x}{+x^2 - 6x + 3}$$

Ⓝ $(7x^2 + 3x + 9) + (2x^2 + 5x - 2) = 9x^2 + 8x + 7$

Ⓤ $(-3x^2 + x - 7) + (8x^2 - 4x - 4) = 5x^2 - 3x - 11$

Ⓙ $(6x^3 + 2x^2 - 3x) + (3x^3 - 10x^2 - x) = 9x^3 - 8x^2 - 4x$

Ⓣ $(-4x^3 + 6x + 1) + (5x^2 - x - 12) = -4x^3 + 5x^2 + 5x - 11$

ⓐ $(9x^3 - x^2 + 8) + (-9x^3 + 2x^2 + 3x) = x^2 + 3x + 8$

Ⓢ $(2x^4 + 5x^2 - 11) + (-6x^4 - 7x^2 + 1) = -4x^4 - 2x^2 - 10$

Ⓝ $(-4x^4 + 3x^3 - 7x^2 - x) + (-9x^3 + 7x^2 - 5x - 1) = -4x^4 - 6x^3 - 6x^2 - 6x - 1$

Ⓙ $(4x^2 + 3xy - y^2) + (x^2 - 8xy - 2y^2) = 5x^2 - 5xy - 3y^2$

ⓐ $(2x^2y - xy^2) + (6x^2y + 7xy^2) = 8x^2y + 6xy^2$

Ⓣ $(x^3y + 3x^2y^2 + 2xy^3) + (2x^3y - 9x^2y^2 - xy^3) = 3x^3y - 6x^2y^2 + xy^3$

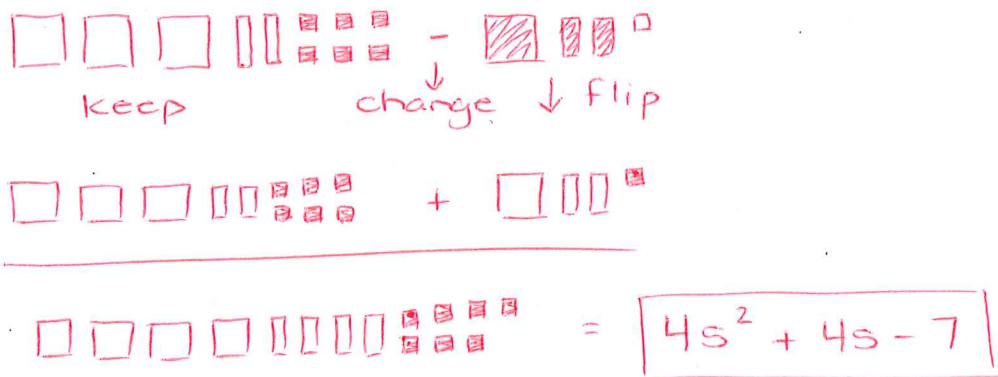
I	T	S	J	U	S	T	A	Z	O	I	T	O	Z
$8x - 11$	$x^2 - 6x + 3$	$8x^2y - 3xy^2$	$5x^2 - 5xy - 3y^2$	$5x^2 - 3x - 11$	$-4x^4 - 2x^2 - 10$	$-4x^3 + 5x^2 - 3x - 1$	$8x^2y + 6xy^2$	$5x^2 - 5xy - 11$	$9x^2 + 8x + 7$	$x^2 + 3x + 8$	$3x^3y - 6x^2y^2 + xy^3$	$9x^3 - 8x^2 - 4x$	$9x^2 - 2x + 8$
$-4x^3 + 5x^2 + 5x - 11$	$x^2 - 6x + 3$	$8x^2y - 3xy^2$	$5x^2 - 5xy - 3y^2$	$5x^2 - 3x - 11$	$-4x^4 - 2x^2 - 10$	$-4x^3 + 5x^2 - 3x - 1$	$8x^2y + 6xy^2$	$5x^2 - 5xy - 11$	$9x^2 + 8x + 7$	$x^2 + 3x + 8$	$3x^3y - 6x^2y^2 + xy^3$	$9x^3 - 8x^2 - 4x$	$9x^2 - 2x + 8$
$-4x^4 - 6x^3 - 6x^2 - 6x - 1$	$x^2 - 6x + 3$	$8x^2y - 3xy^2$	$5x^2 - 5xy - 3y^2$	$5x^2 - 3x - 11$	$-4x^4 - 2x^2 - 10$	$-4x^3 + 5x^2 - 3x - 1$	$8x^2y + 6xy^2$	$5x^2 - 5xy - 11$	$9x^2 + 8x + 7$	$x^2 + 3x + 8$	$3x^3y - 6x^2y^2 + xy^3$	$9x^3 - 8x^2 - 4x$	$9x^2 - 2x + 8$
$-4x^4 - 6x^3 - 6x^2 - 6x - 1$	$x^2 - 6x + 3$	$8x^2y - 3xy^2$	$5x^2 - 5xy - 3y^2$	$5x^2 - 3x - 11$	$-4x^4 - 2x^2 - 10$	$-4x^3 + 5x^2 - 3x - 1$	$8x^2y + 6xy^2$	$5x^2 - 5xy - 11$	$9x^2 + 8x + 7$	$x^2 + 3x + 8$	$3x^3y - 6x^2y^2 + xy^3$	$9x^3 - 8x^2 - 4x$	$9x^2 - 2x + 8$
$-4x^4 - 6x^3 - 6x^2 - 6x - 1$	$x^2 - 6x + 3$	$8x^2y - 3xy^2$	$5x^2 - 5xy - 3y^2$	$5x^2 - 3x - 11$	$-4x^4 - 2x^2 - 10$	$-4x^3 + 5x^2 - 3x - 1$	$8x^2y + 6xy^2$	$5x^2 - 5xy - 11$	$9x^2 + 8x + 7$	$x^2 + 3x + 8$	$3x^3y - 6x^2y^2 + xy^3$	$9x^3 - 8x^2 - 4x$	$9x^2 - 2x + 8$

Example #2: Subtract the following polynomials using different methods.

$$(3s^2 + 2s - 6) - (-s^2 - 2s + 1)$$

First, rewrite the subtraction as addition (keep, change, flip). Then add.

□ Algebra Tiles



□ Symbolically/Horizontally

$$\begin{aligned} & (3s^2 + 2s - 6) - (-s^2 - 2s + 1) \\ &= (3s^2 + 2s - 6) + (s^2 + 2s - 1) \\ &= 3s^2 + s^2 + 2s + 2s - 6 - 1 \\ &= \boxed{4s^2 + 4s - 7} \end{aligned}$$

□ Vertically

$$\begin{array}{r} 3s^2 + 2s - 6 \\ + \quad s^2 + 2s - 1 \\ \hline 4s^2 + 4s - 7 \end{array}$$

Practice

3. Write the opposite of each term.

a) -9 : 9 b) $3r$: $-3r$ c) $-2s^2$: $+2s^2$ d) t : $-t$

4. Subtract.

a) $(4p + 1) - (p + 10)$
 $=$ $4p + 1$ $- (p + 10)$

$=$ $4p + 1 +$ $-p - 10$
 $=$ $4p - p + 1 - 10$
 $=$ $3p - 9$

Remove the brackets from the first term.

The opposite of p is: $-p$

The opposite of 10 is: -10

Add the opposites.

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

b) $(3h^2 + 5h - 4) - (h^2 - 4h + 6)$
 $=$ $3h^2 + 5h - 4 - (h^2 - 4h + 6)$
 $=$ $3h^2 + 5h - 4 + (-h^2 + 4h - 6)$
 $=$ $3h^2 + 5h - 4 - h^2 + 4h - 6$
 $=$ $3h^2 - h^2 + 5h + 4h - 4 - 6$
 $=$ $2h^2 + 9h - 10$

Remove the brackets from the first term.

Add the opposites.

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

c) $(4q^2 + 3) - (3q - q^2 + 3)$
 $=$ $4q^2 + 3 + (-3q + q^2 - 3)$
 $=$ $4q^2 + 3 - 3q + q^2 - 3$
 $=$ $4q^2 + q^2 - 3q + 3 - 3$
 $=$ $5q^2 - 3q$

5. Check each solution. Identify any errors and correct them.

a) $(7x^2 + 3x + 7) - (3x^2 - 4)$
 $=$ $7x^2 + 3x + 7 - 3x^2 - 4$
 $=$ $7x^2 - 3x^2 + 3x + 7 - 4$
 $=$ $4x^2 + 3x + 3$

$(7x^2 + 3x + 7) - (3x^2 - 4)$
 $=$ $7x^2 + 3x + 7 - 3x^2 + 4$
 $=$ $7x^2 - 3x^2 + 3x + 7 + 4$
 $=$ $4x^2 + 3x + 11$

b) $(3a^2 - 2a + 4) - (2a^2 + 3)$
 $=$ $3a^2 - 2a + 4 - 2a^2 - 3$
 $=$ $3a^2 - 2a^2 - 2a + 4 - 3$
 $=$ $a^2 + 2a - 3$

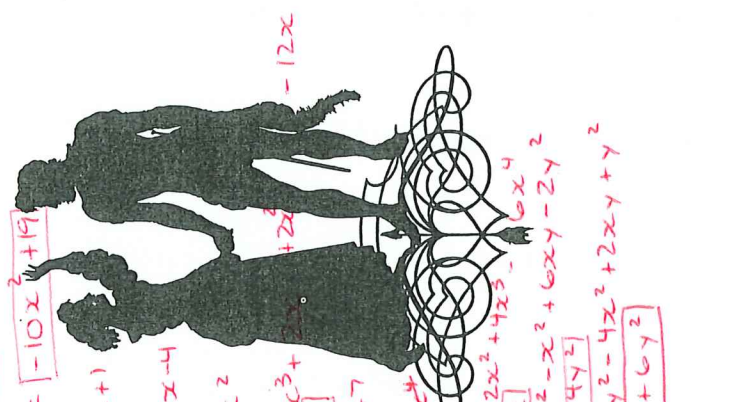
$(3a^2 - 2a + 4) - (2a^2 + 3)$
 $=$ $3a^2 - 2a + 4 - 2a^2 - 3$
 $=$ $3a^2 - 2a^2 - 2a + 4 - 3$
 $=$ $a^2 - 2a + 1$

Daffynition Decoder

1. Romantic: A N I T A L I A N I N S E C T
 11 13 8 12 11 1 8 11 13 8 13 10 3 5 12
2. American: A H A P P Y C O N T A I N E R
 11 2 11 9 9 6 5 7 13 12 11 8 13 3 4

For each exercise below, subtract the second polynomial from the first. Find your answer in the answer column and notice the letter next to it. Each time the exercise number appears in the code, write this letter above it. Keep working and you will decode the "de-fun-itions."

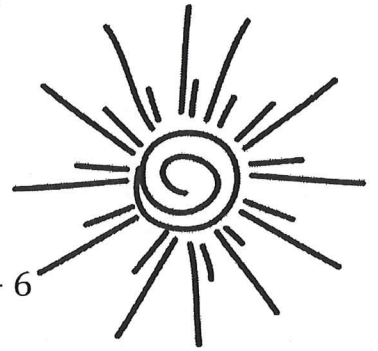
- 1 $(7x + 4) - (2x + 9) = 7x + 4 - 2x - 9 = 5x - 5$
- 2 $(3x + 12) - (5x - 6) = 3x + 12 - 5x + 6 = -2x + 18$
- 3 $(-4x^2 + 10) - (6x^2 - 9) = -4x^2 + 10 - 6x^2 + 9 = -10x^2 + 19$
- 4 $(2x^2 + 3x + 8) - (x^2 + 5x - 1) = 2x^2 + 3x + 8 - x^2 - 5x + 1 = x^2 - 2x + 9$
- 5 $(-x^2 + 9x - 2) - (9x^2 - 4x + 4) = -x^2 + 9x - 2 - 9x^2 + 4x - 4 = -10x^2 + 13x - 6$
- 6 $(3x^2 + 7x + 1) - (8 + 5x + x^2) = 3x^2 + 7x + 1 - 8 - 5x - x^2 = 2x^2 + 2x - 7$
- 7 $(4x^3 + 6x^2 - 8x) - (x^3 - 2x^2 + 12x) = 4x^3 + 6x^2 - 8x - x^3 + 2x^2 - 12x = 3x^3 + 8x^2 - 20x$
- 8 $(x^3 + 2x^2 + 5x) - (3x^2 - x - 7) = x^3 + 2x^2 + 5x - 3x^2 + x + 7 = x^3 - x^2 + 6x + 7$
- 9 $(x^4 + 8x^2 - 1) - (x^2 - 3x^3 + x^4) = x^4 + 8x^2 - 1 - x^2 + 3x^3 - x^4 = 3x^3 + 7x^2 - 1$
- 10 $(5x^4 - 2x^2) - (3x - 2x^2 - 4x^3 + 6x^4) = 5x^4 - 2x^2 - 3x + 2x^2 + 4x^3 - 6x^4 = -x^4 + 4x^3 - 3x$
- 11 $(3x^2 + 7xy - 2y^2) - (x^2 - 6xy + 2y^2) = 3x^2 + 7xy - 2y^2 - x^2 + 6xy - 2y^2 = 2x^2 + 13xy - 4y^2$
- 12 $(-x^2 - 9xy + 5y^2) - (4x^2 - 2xy - y^2) = -x^2 - 9xy + 5y^2 - 4x^2 + 2xy + y^2 = -5x^2 - 7xy + 6y^2$
- 13 $(4x^2y - 3xy^2) - (3x^2y - 8xy^2) = 4x^2y - 3xy^2 - 3x^2y + 8xy^2 = x^2y + 5xy^2$



Answers:

- (M) $-x^4 + 4x^3 - 7x^2$
 (S) $-x^4 + 4x^3 - 3x$
 (U) $3x^3 + 5x^2 + 7$
 (L) $5x - 5$
 (E) $-10x^2 + 19$
 (F) $2x^2 + 2x - 19$
 (C) $-10x^2 + 13x - 6$
 (H) $-2x + 18$
 (T) $-5x^2 - 7xy + 6y^2$
 (O) $3x^3 + 8x^2 - 20x$
 (P) $3x^3 + 7x^2 - 1$
 (R) $x^2 - 2x + 9$
 (A) $2x^2 + 13xy - 4y^2$
 (N) $x^2y + 5xy^2$
 (Y) $2x^2 + 2x - 7$
 (B) $-5x^2 - 6xy + 7y^2$
 (I) $x^3 - x^2 + 6x + 7$

What season is it when you are on a trampoline?



Simplify

1. $(3x^3 + 4 - 2x) + (2x^2 - x + 6)$

$3x^3 + 2x^2 - 3x + 10$

2. $(5x^2 + 15x - 4) - (3x^2 - 1 + 7x)$

$5x^2 + 15x - 4 - 3x^2 + 1 - 7x$
 $2x^2 + 8x - 3$

3. $(11 + 3x^3) + (4x^2 - 2) + 3x$

$3x^3 + 4x^2 + 3x + 9$

4. $(x^3 - 4x + 7) - (-2x^3 + 6x + 1)$

$x^3 - 4x + 7 + 2x^3 - 6x - 1$
 $3x^3 - 10x + 6$

5. $(14x^2 + 8) + (4x - 2) - (12x^2 - 1)$

$14x^2 + 8 + 4x - 2 - 12x^2 + 1$
 $2x^2 + 4x + 7$

6. $4x - (-4x^3 + 11) - (2x + x^3) + 7x$

$4x + 4x^3 - 11 - 2x - x^3 + 7x$
 $3x^3 + 9x - 11$

7. $(6x^2 - 7x + 2) + (15x + 5 - 4x^2)$

$2x^2 + 8x + 7$

8. $(-2x^3 + x^2 + x - 3) + (5x^3 + x^2 - x)$

$3x^3 + 2x^2 - 3$

9. $(15x^2 + x^3 + 7) - (6x^2 - 2x^3) - 7x^2$

$15x^2 + x^3 + 7 - 6x^2 + 2x^3 - 7x^2$
 $3x^3 + 2x^2 + 7$

Answers

~~E.~~ $3x^3 - 10x + 6$

~~G.~~ $3x^3 + 2x^2 - 3$

~~I.~~ $3x^3 + 2x^2 - 3x + 10$

~~M.~~ $3x^3 + 9x - 11$

~~N.~~ $2x^2 + 8x + 7$

~~P.~~ $3x^3 + 2x^2 + 7$

~~R.~~ $3x^3 + 4x^2 + 3x + 9$

~~S.~~ $2x^2 + 4x + 7$

~~T.~~ $2x^2 + 8x - 3$



5	9	3	1	7	8	2	1	6	4
S	P	R	I	N	G	T	I	M	E