Name: KEY
Date: _____

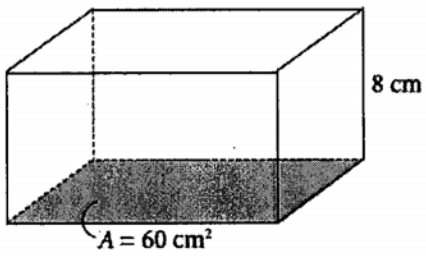
Math 8
Lesson M3 ~ Calculating Volume of Right Rectangular Prisms, Right Triangular Prisms, & Cylinders

- Volume = Area of base x Height
  - Area $_{rectangle}$ = length x width
  - Area $_{triangle}$ = length x width ÷ 2
  - Area $_{circle}$ = π x r^2

- Basic:

1. The area of the base and the height are shown on each rectangular prism. Determine the volume of each prism.

a)



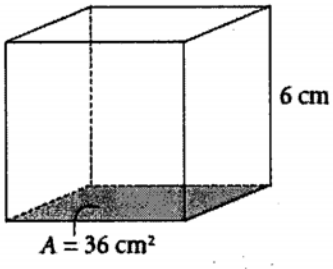
$V = Ah$

$= 60 \times 8$

$= 480$

The volume is 480 cm³.

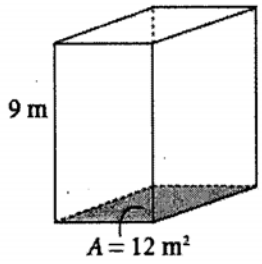
b)



The volume is 216 cm³.

36 x 6

c)

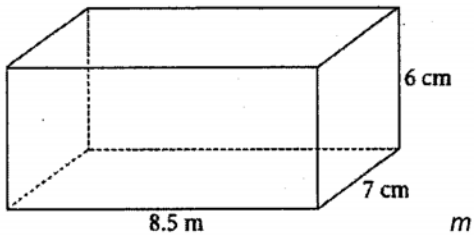


The volume is 108 m³.

12 x 9

2. Determine the volume of each prism.

a)



m

m

$A = 8.5 \times 7$

$= 59.5$

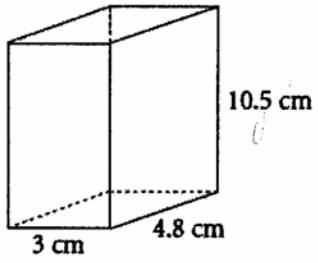
$V = Ah$

$= 59.5 \times 6$

$= 357$

The volume is 357 m³.

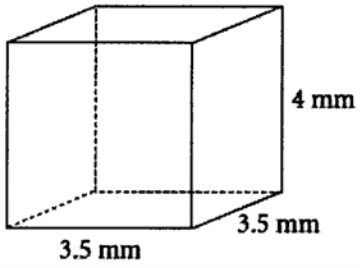
b)



The volume is 151.2 cm^3.

$3 \times 4.8 \times 10.5$

c)

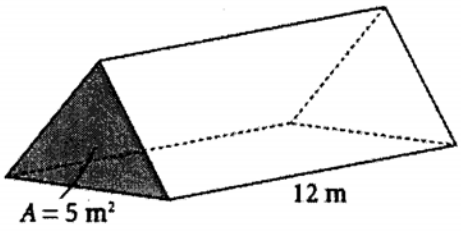


The volume is 49 mm^3.

$3.5 \times 3.5 \times 4$

1. The area of the base and the length of each prism are shown. Calculate the volume of each prism.

a)



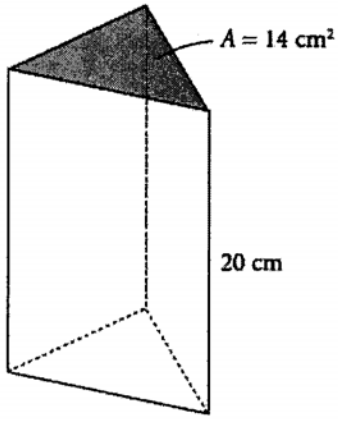
$V = A\ell$

$= 5 \times 12$

$= 60$

The volume is 60 m^3.

b)



The volume is 280 cm^3.

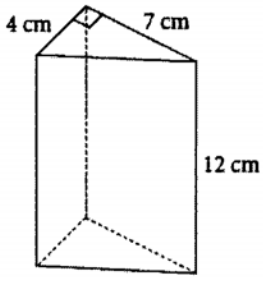
14×20

2. Determine the volume of each prism.

a) $A = \frac{1}{2}bh$ or $\frac{bh}{2}$

$= \frac{1}{2} \times 4 \times 7$

$= 14\ cm^2$



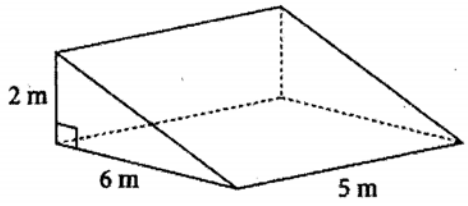
$V = A\ell$

$= 14 \times 12$

$= 168$

The volume is 168 cm^3.

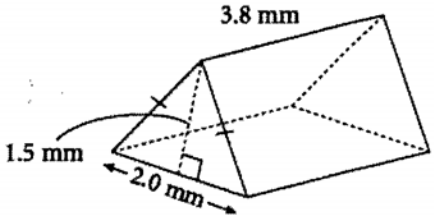
b)



The volume is $30\ m^3$.

$\frac{2 \times 6}{2} \times 5$

c)

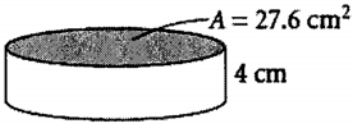


The volume is $5.7\ mm^3$.

$\frac{2 \times 1.5}{2} \times 3.8$

1. The base area and height of each cylinder are given. Calculate the volume, to the nearest cubic unit.

a)

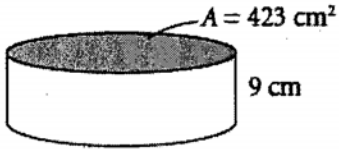


Volume of a cylinder = base area × height

= 27.6 × 4

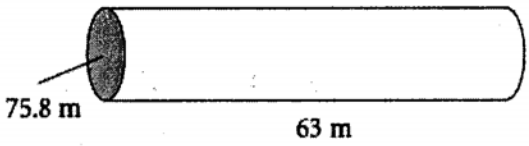
= 110.4 cm³

b)



$423 \times 9 = 3807\ cm^3$

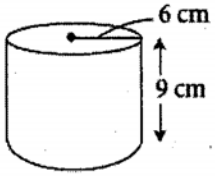
c)



$75.8 \times 63 = 4775.4\ m^3$

2. Calculate the volume of each cylinder, to the nearest cubic unit.

a)



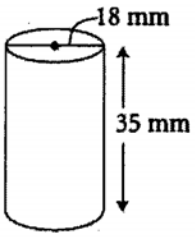
$V = \pi r^2 h$

$= \pi \times 6^2 \times 9$

$\doteq 1017.36$ (or 972)

The volume of the cylinder is 1017, to the nearest cubic cm.

b) r = 9 mm

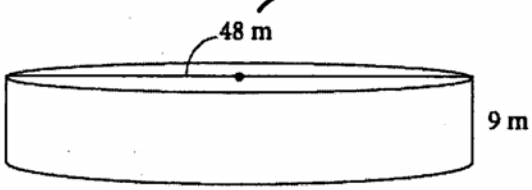


$V = \pi \times 9^2 \times 35$

$= 8901.9 \text{ mm}^3$

(or 8505)

c) r = 24 m



$V = \pi \times 24^2 \times 9$

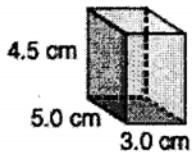
$= 16\,277.76 \text{ m}^3$

(or 15 552)

- Intermediate:

7. Find the volume of each rectangular prism.

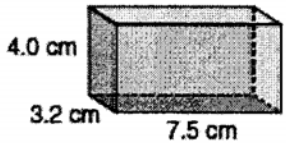
a)



$V = 3 \times 5 \times 4.5$

$= 67.5 \text{ cm}^3$

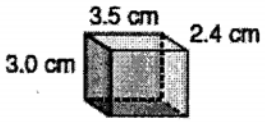
b)



$V = 3.2 \times 7.5 \times 4$

$= 96 \text{ cm}^3$

c)



$V = 3 \times 3.5 \times 2.4$

$= 25.2 \text{ cm}^3$

9. Each dogsled team that enters the Iditarod has a portable doghouse for each sled dog. Two mushers are comparing the sizes of their doghouses. Each of Rick's doghouses is 94 cm by 63 cm by 71 cm. Each of Susan's doghouses is 109 cm by 71 cm by 81 cm.

a) What is the volume of each doghouse?

b) About how many times as great as the volume of Rick's doghouse is the volume of Susan's doghouse?

a) Rick's: $V = 94 \times 63 \times 71$

$= 420\,462 \text{ cm}^3$

Susan's: $V = 109 \times 71 \times 81$

$= 626\,859 \text{ cm}^3$

b) $\frac{626\,859}{420\,462} = 1.49 \approx 1.5$

→ Susan's doghouses are 1.5 times bigger than Rick's doghouses

11. Large trucks often tow trailers that are shaped like right rectangular prisms. A standard trailer is 2.74 m by 2.43 m by 6.1 m.

a) What is the greatest volume of cargo a standard trailer can hold?

b) How many trailers would it take to transport 100 m^3 of goods? What assumptions do you make?

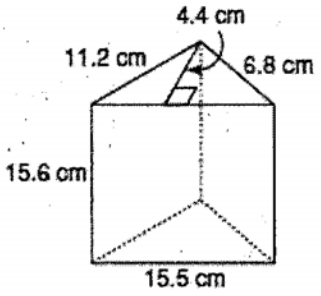
a) $V = 2.74 \times 2.43 \times 6.1$
$= 40.61502\ m^3$

b) $\frac{100}{40.6} = 2.463$

→ It would take 3 trailers to transport 100 m^3 of goods, assuming there are no weight restrictions and that the items all fit (without much space in between).

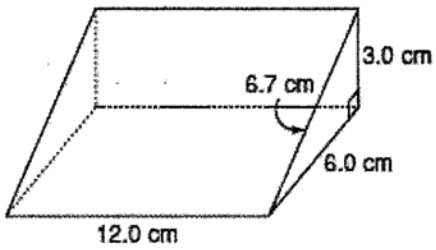
6. Find the volume of each prism.

a)



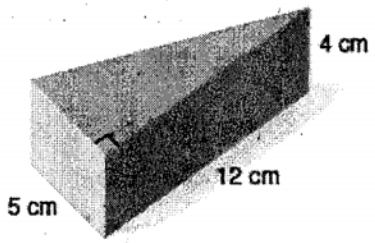
6.a) $V = \frac{15.5 \times 4.4}{2} \times 15.6$
$= 531.96\ cm^3$

b)



b) $V = \frac{6 \times 3}{2} \times 12$
$= 108\ cm^3$

10. Chico has a wedge of cheddar cheese. He plans to serve the cheese as an appetizer before dinner.



a) What volume of cheese does Chico have?

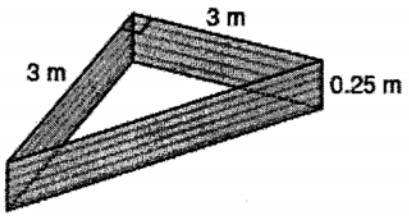
b) Suppose each person eats 20 cm^3 of cheese. How many people will the cheese serve?

a) $V = \frac{5 \times 12}{2} \times 4$
$= 120\ cm^3$

b) $\frac{120}{20} = 6$

→ The cheese will serve 6 people.

12. Jackie uses this form to build a concrete pad.



a) How much concrete will Jackie need to mix to fill the form?

b) Suppose Jackie increases the lengths of the equal sides of the form from 3 m to 6 m.
How much more concrete will Jackie need to mix?
Include a diagram.

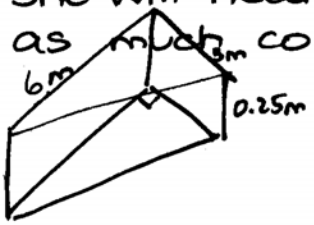
a) $V = \frac{3 \times 3}{2} \times 0.25$

$= 1.125\ m^3$

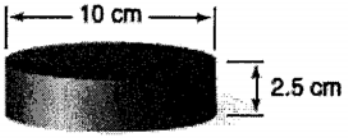
b) $V = \frac{3 \times 6}{2} \times 0.25$

$= 2.25\ m^3$

→ She will need twice as much concrete.



8. A hockey puck is a solid piece of rubber with the dimensions shown.
How much rubber is used to make a hockey puck?



$V = \pi \times 5^2 \times 2.5$

$V = 196.25\ cm^3$

10. Kari has 125 mL of water. She wants to pour it into one of these cylindrical bottles. Which bottle will hold all the water? How do you know?
Bottle A: $d = 7$ cm, $h = 3$ cm
Bottle B: $r = 2$ cm, $h = 6$ cm
Bottle C: $r = 3.5$ cm, $h = 7$ cm
Bottle D: $d = 3$ cm, $h = 4$ cm

A: $V = \pi \times 3.5^2 \times 3$
$= 115.4$

B: $V = \pi \times 2^2 \times 6$
$= 75.36$

C: $V = \pi \times 3.5^2 \times 7$
$= 269.3$

D: $V = \pi \times 1.5^2 \times 4$
$= 28.26$

→ Bottle C is the only one that is big enough (volume is greater than 125mL)

14. A farmer has 3 cylindrical containers to hold feed. Each container has radius 91 cm and height 122 cm. What is the total volume of the three containers? How did you find out?

$V = \pi \times 91^2 \times 122$
$= 3172\ 285.48$
$\times 3$
$= 9\ 516\ 856.44\ cm^3$

- Advanced: please complete Intermediate questions first.

10. Suppose a milk carton is 10 cm wide and 10 cm long. How tall must the carton be to hold 1 L of milk?

Recall 1 cm^3 = 1 mL.

1L = 1000 mL

10 × 10 × 10 = 1000

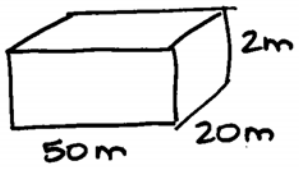
so height = 10 cm

12. A rectangular swimming pool is to be filled with water. The pool has a uniform depth of 2 m and is surrounded by a wooden deck. The pool is 20 m wide and 50 m long. How much water is needed in each case?

a) The pool is filled to the level of the deck.

b) The pool is filled to within 20 cm of the level of the deck.

c) The pool is half filled.



a) $V = 50 \times 20 \times 2 = 2000\ m^3$

b) 2 m − 20 cm = 2 − 0.2 = 1.8 m

$V = 50 \times 20 \times 1.8 = 1800\ m^3$

c) $V = 50 \times 20 \times 1 = 1000\ m^3$

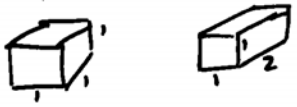
15. Sketch a right rectangular prism. Label its dimensions.
What do you think happens to the volume of the prism when:

a) its length is doubled?

b) its length and width are doubled?

c) its length, width, and height are doubled?

Investigate to find out.
Show your work.
Will the results be true for all rectangular prisms?
Why do you think so?



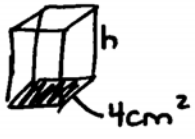
a) $V = 1 \times 1 \times 1 = 1\ cm^3$
$V = 1 \times 1 \times 2 = 2\ cm^3$ } doubled

b) $V = 1 \times 2 \times 2 = 4\ cm^3$ } quadrupled

c) $V = 2 \times 2 \times 2 = 8\ cm^3$ } eight times larger

→ Yes this will be true for all rectangular prisms

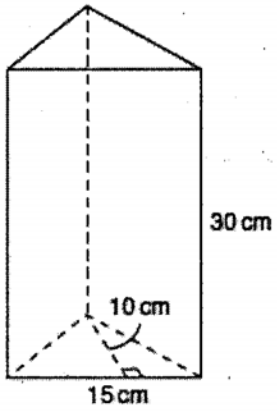
8. The volume of a right triangular prism is 30 cm^3. Each triangular face has area 4 cm^2.
How long is the prism?



$V = 4 \times h$

$\frac{30}{4} = \frac{4 \times h}{4}$ $h = 7.5$ cm

14. a) Find the volume of this prism.



a) $V = \frac{15 \times 10}{2} \times 30 = 2250\ cm^3$

b) Suppose the prism contains 1350 mL of water. What is the depth of the water?

b) $1350 = \frac{10 \times 15}{2} \times h$.

$\frac{1350}{75} = \frac{75 \times h}{75}$

$1350\ cm^3$ $h = 18$ cm.

c) What percent of the volume of the prism is water?

c) $\frac{1350}{2250} = 60\%$

11. Assessment Focus Frozen apple juice comes in cylindrical cans. A can is 12 cm high with radius 3.5 cm.

a) What is the capacity of the can?

b) What happens to the capacity of the can if the dimensions of the radius and height are switched?

a) $V = \pi \times 3.5^2 \times 12$

$= 461.58\ cm^3$

$= 461.58$ mL.

b) $V = \pi \times 12^2 \times 3.5$

$= 4521.6\ cm^3$

$= 4521.6$ mL

17. Take It Further

A concrete column in a parkade is cylindrical. The column is 10 m high with diameter 3.5 m.

a) What is the volume of concrete in one column?

b) There are 127 columns in the parkade. What is the total volume of concrete?

c) Suppose the concrete in part a is made into a cube. What would the dimensions of the cube be?

a) $V = \pi \times \left(\frac{3.5}{2}\right)^2 \times 10$

$= 96.1625\ m^3$

b) 96.1625×127

$= 12\ 212.6375\ m^3$

c) $l \times l \times l = 96.1625$

$\sqrt[3]{l^3} = \sqrt[3]{96.1625}$

$l = 4.58$ m