KEY

Pre-Calculus 11
Lesson 9.2 ~ Linear & Quadratic Inequalities in One Variable

In this chapter, we will look at linear and quadratic **inequalities** instead of linear and quadratic **equations**. Inequalities look the same as equations, but instead of an "=" sign, there is one of the following four signs: <, ≤, >, or ≥.

< means **less than** **>** means **greater than**
≤ means **less than or equal to** **≥** means **greater than or equal to**

< and **>** are represented by an **open circle** on a number line and **exclude** the x-intercept(s).
≤ and **≥** are represented by a **closed circle** on a number line and **include** the x-intercept(s).

Linear inequalities in one variable can be written in one of four forms:

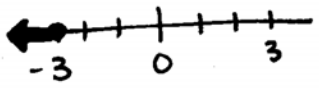
- $mx + b < 0$
- $mx + b \leq 0$
- $mx + b > 0$
- $mx + b \geq 0$

Quadratic inequalities in one variable can be written in one of four forms:

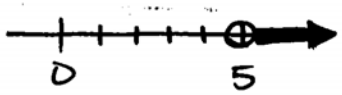
- $ax^2 + bx + c < 0$
- $ax^2 + bx + c \leq 0$
- $ax^2 + bx + c > 0$
- $ax^2 + bx + c \geq 0$

Example #1: Graph the solution set for each linear inequality.

a) $x \leq -3$



b) $5 < x \Rightarrow x > 5$



Example #2: Solve each linear inequality.

a) $3x + 4 > 5x + 2$

$-3x \quad -3x$

$4 > 2x + 2$
$-2 \quad -2$

$\frac{2}{2} > \frac{2x}{2}$

$1 > x$

or

$\boxed{x < 1}$

b) $7n - 2(n + 5) \leq 3n - 16$

$7n - 2n - 10 \leq 3n - 16$

$5n - 10 \leq 3n - 16$
$-3n \quad -3n$

$2n - 10 \leq -16$
$+10 \quad +10$

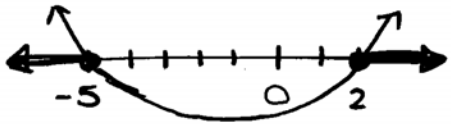
$\frac{2n}{2} \leq \frac{-6}{2}$

$\boxed{n \leq -3}$

Example #3: Solve $x^2 + 3x - 10 \geq 0$.

$(x+5)(x-2) \geq 0$

$x = -5 \quad , \quad 2$



test regions:

- $x \leq -5$
  $(-6)^2 + 3(-6) - 10 \geq 0$?
  $8 \geq 0$ ✓
- $-5 \leq x \leq 2$
  $0^2 + 3(0) - 10 \geq 0$?
  $-10 \geq 0$ ✗
- $x \geq 2$
  $3^2 + 3(3) - 10 \geq 0$?
  $8 \geq 0$ ✓

$\boxed{x \leq -5 \ , \ x \geq 2}$

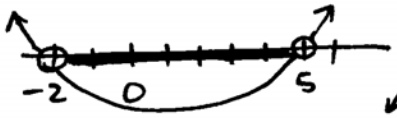
Example #4: Solve $-x^2 + 3x + 10 > 0$. Check your solution.

$+x^2 - 3x - 10 \qquad +x^2 - 3x - 10$

$0 > x^2 - 3x - 10$

$0 > (x-5)(x+2)$

$x = 5 \ , \ -2$

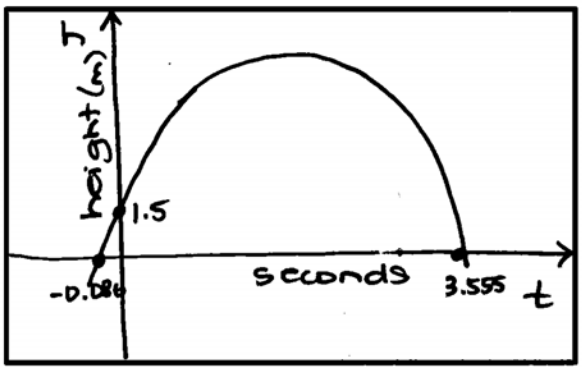


test regions:

- $x < -2$
  $-(-3)^2 + 3(-3) + 10 > 0$?
  $-8 > 0$ ✗
- $-2 < x < 5$
  $-0^2 + 3(0) + 10 > 0$?
  $10 > 0$ ✓
- $x > 5$
  $-6^2 + 3(6) + 10 > 0$?
  $-8 > 0$ ✗

$\boxed{-2 < x < 5}$

Example #5: Suppose a baseball is thrown from a height of 1.5 m. The inequality $-4.9t^2 + 17t + 1.5 > 0$ models the time, t, in seconds, that the baseball is in flight. During what time interval is the baseball in flight?



$t = \dfrac{-17 \pm \sqrt{17^2 - 4(-4.9)(1.5)}}{2(-4.9)}$

$t = \dfrac{-17 \pm \sqrt{318.4}}{-9.8}$

$t = -0.086 \ , 3.555$

$\boxed{0 < t < 3.555}$

Baseball was in flight between 0s and 3.555s