KEY

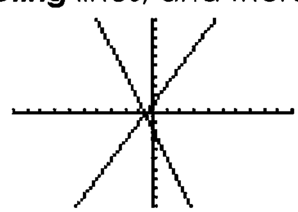
Lesson 7.6 ~ Properties of Systems of Linear Equations

All of the linear systems that you have worked with so far have had exactly one solution. But this is not always the case. A linear system can have:

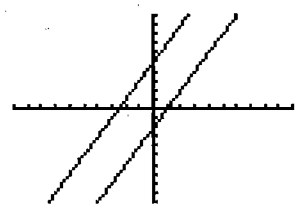
- No solution
- One solution
- Infinitely many solutions

Steps:

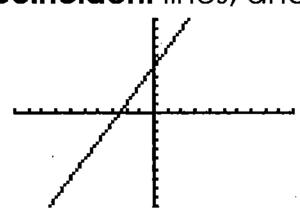
1. Solve for y ($y = mx + b$).
2. Compare slopes (m_1 and m_2) and y-intercepts (b_1 and b_2).
    i. If $m_1 \neq m_2$, you have two **intersecting** lines, and therefore there is **one** solution (where they intersect).



    ii. If $m_1 = m_2$ but $b_1 \neq b_2$, you have **parallel** lines, and therefore there is **no** solution.



    iii. If $m_1 = m_2$ and $b_1 = b_2$, you have **coincident** lines, and therefore there are **infinitely** many solutions.



**Equations can also be graphed to determine the number of solutions.

**Short cut for finding slope and y-intercept if equation is written in general form:

$$Ax + By + C = 0$$

$$By = -Ax - C$$

$$y = \frac{-A}{B}x - \frac{C}{B}$$

So…the slope is $m = \frac{-A}{B}$ and the y-intercept is $b = \frac{-C}{B}$.

Example #1: Determine the number of solutions of the following system.

$$-7x + y = 10 \longrightarrow -\cancel{7x} + y = 10$$
$$+\cancel{7x} \quad +7x$$
$$y = 7x + 10$$

$$-\cancel{14x} + 2y = 20$$
$$+\cancel{14x} \quad +14x$$
$$\frac{\cancel{2}y}{\cancel{2}} = \frac{14x}{2} + \frac{20}{2}$$
$$y = 7x + 10$$

★ Slopes & y-intercepts are the same, so these are coincidental lines and have infinitely many solutions

Example #2: Determine the number of solutions of the following system.

$$2x - 5y = 15 \longrightarrow \cancel{2x} - 5y = 15$$
$$-\cancel{2x} \quad -2x$$
$$\frac{-\cancel{5}y}{-\cancel{5}} = \frac{-2x}{-5} + \frac{15}{-5}$$
$$y = \frac{2}{5}x - 3$$

$$\cancel{2x} - 5y = -5$$
$$-\cancel{2x} \quad -2x$$
$$\frac{-\cancel{5}y}{-\cancel{5}} = \frac{-2x}{-5} \frac{-5}{-5}$$
$$y = \frac{2}{5}x + 1$$

★ Slopes are the same but y-intercepts are different, so these lines are parallel and there is no solution

Example #3: Determine the number of solutions of the following system.

$$2x - 3y = 3 \longrightarrow \cancel{2x} - 3y = 3$$
$$-\cancel{2x} \quad -2x$$
$$\frac{-\cancel{3}y}{-\cancel{3}} = \frac{-2x}{-3} + \frac{3}{-3}$$
$$y = \frac{2}{3}x - 1$$

$$\cancel{5x} + y = 16$$
$$-\cancel{5x} \quad -5x$$
$$y = -5x + 16$$

★ Slopes are different so these lines will cross once and have one solution

Example #4: Consider the equation $x - 2y - 8 = 0$. Write a second equation to form a linear system with

a) No solution

$x - 2y - 8 = 0$

↳ changing C will affect the y-intercept

$\boxed{x - 2y + 5 = 0}$

↳ now the lines will have the same slope and different y-intercepts

b) Infinitely many solutions

$(x - 2y - 8 = 0) \times 2 \rightarrow \boxed{2x - 4y - 16 = 0}$

↳ multiply or divide by any factor; now the lines will have the same slope and y-intercept

c) One solution

$x - 2y - 8 = 0$

↳ change x and/or y coefficients to change the slope

$\boxed{3x - 2y - 8 = 0}$