**Lesson 7.6 ~ Properties of Systems of Linear Equations**

All of the linear systems that you have worked with so far have had exactly one solution. But this is not always the case. A linear system can have:

- No solution
- One solution
- Infinitely many solutions

**Steps:**

1. Solve for \( y \) (\( y = mx + b \)).
2. Compare slopes (\( m_1 \) and \( m_2 \)) and \( y \)-intercepts (\( b_1 \) and \( b_2 \)).
   i. If \( m_1 \neq m_2 \), you have two intersecting lines, and therefore there is one solution (where they intersect).
   ![Intersecting Lines](image)
   
   ii. If \( m_1 = m_2 \) but \( b_1 \neq b_2 \), you have parallel lines, and therefore there is no solution.
   ![Parallel Lines](image)

   iii. If \( m_1 = m_2 \) and \( b_1 = b_2 \), you have coincident lines, and therefore there are infinitely many solutions.
   ![Coincident Lines](image)

**Equations can also be graphed to determine the number of solutions.**

**Short cut for finding slope and \( y \)-intercept if equation is written in general form:**

\[
Ax + By + C = 0
\]

\[
By = -Ax - C
\]

\[
y = -\frac{A}{B}x - \frac{C}{B}
\]

So...the slope is \( m = -\frac{A}{B} \) and the \( y \)-intercept is \( b = -\frac{C}{B} \).
Example #1: Determine the number of solutions of the following system.

\[-7x + y = 10 \quad \rightarrow \quad -7x + y = 10\]
\[-14x + 2y = 20 \quad + 7x \quad + 7x\]
\[+14x \quad + 14x\]
\[2y = 14x + 20\]
\[\frac{2y}{2} = \frac{14x + 20}{2}\]
\[y = 7x + 10\]

* Slopes & y-intercepts are the same, so these are coincidental lines and have infinitely many solutions.

Example #2: Determine the number of solutions of the following system.

\[2x - 5y = 15 \quad \rightarrow \quad 2x - 5y = 15\]
\[2x - 5y = -5 \quad -2x \quad -2x\]
\[-2x \quad -2x\]
\[-5y = -2x - 5\]
\[\frac{-5y}{-5} = \frac{-2x - 5}{-5}\]
\[y = \frac{2}{5}x - 3\]

* Slopes are the same but y-intercepts are different, so these lines are parallel and there is no solution.

Example #3: Determine the number of solutions of the following system.

\[2x - 3y = 3 \quad \rightarrow \quad 2x - 3y = 3\]
\[5x + y = 16 \quad -2x \quad -2x\]
\[-5x \quad -5x\]
\[-3y = -2x + 3\]
\[\frac{-3y}{-3} = \frac{-2x + 3}{-3}\]
\[y = \frac{2}{3}x - 1\]

* Slopes are different so these lines will cross once and have one solution.
Example #4: Consider the equation \( x - 2y - 8 = 0 \). Write a second equation to form a linear system with

a) No solution

\[
x - 2y - 8 = 0
\]

\( \Rightarrow \) changing \( C \) will affect the \( y \)-intercept

\[
x - 2y + 5 = 0
\]

\( \Rightarrow \) now the lines will have the same slope and different \( y \)-intercepts

b) Infinitely many solutions

\[
(x - 2y - 8 = 0) \times 2 \rightarrow 2x-4y-16 = 0
\]

\( \Rightarrow \) multiply or divide by any factor; now the lines will have the same slope and \( y \)-intercept

c) One solution

\[
x - 2y - 8 = 0
\]

\( \Rightarrow \) change \( x \) and/or \( y \) coefficients to change the slope

\[
3x - 2y - 8 = 0
\]