

Lesson 7.5 ~ Using Elimination to Solve a System of Linear Equations

The third of three methods is elimination (also known as the addition method).

Elimination Steps:

1. One of the variables must have the same coefficient (number in front) in both equations; if not, multiply one or both of the equations to create equal coefficients.
2. Add or subtract the equations to eliminate one of the variables.
3. Solve for the remaining variable.
4. Substitute known value into either original equation.
5. Solve for remaining variable.
6. Check your answer.

Example #1: Solve the linear system and verify the solution.

$$\begin{array}{r} 3x - 5y = -9 \\ + (4x + 5y = 23) \\ \hline 7x + 0y = 14 \end{array}$$

$$\frac{7x}{7} = \frac{14}{7}$$

$$\boxed{x = 2}$$

$$\begin{array}{r} \longrightarrow 3(2) - 5y = -9 \\ 6 - 5y = -9 \\ +9 + 5y \quad +5y + 9 \end{array}$$

$$\frac{15}{5} = \frac{5y}{5}$$

$$\boxed{y = 3}$$

verify: ① $3(2) - 5(3) = -9$

$$\begin{array}{r} 6 - 15 = -9 \\ -9 = -9 \checkmark \end{array}$$

② $4(2) + 5(3) = 23$

$$\begin{array}{r} 8 + 15 = 23 \\ 23 = 23 \checkmark \end{array}$$

Example #2: Solve the linear system and verify the solution.

$$\textcircled{1} (3x + 4y = 29) \times 2 \Rightarrow 6x + 8y = 58$$

$$~~2x - 5y = 19~~$$

$$\textcircled{2} (2x - 5y = -19) \times 3 \Rightarrow -(6x - 15y = -57)$$

$$\underline{0x + 23y = 115}$$

$$\frac{23y}{23} = \frac{115}{23}$$

$$\boxed{y = 5} \rightarrow 3x + 4(5) = 29$$

$$3x + 20 = 29$$

$$\underline{-20 \quad -20}$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$\boxed{x = 3}$$

verify: $\textcircled{1} 3(3) + 4(5) = 29$

$$9 + 20 = 29$$

$$29 = 29 \checkmark$$

$$\textcircled{2} 2(3) = 5(5) - 19$$

$$6 = 25 - 19$$

$$6 = 6 \checkmark$$

Example #3: Solve the linear system and verify the solution.

$$\textcircled{1} \left(\frac{3}{4}x - y = 2\right) \times 4 \Rightarrow 3x - 4y = 8 \Rightarrow 3x - 4y = 8$$

$$\textcircled{2} \left(\frac{1}{8}x + \frac{1}{4}y = 2\right) \times 8 \Rightarrow (x + 2y = 16) \times 2 \Rightarrow + (2x + 4y = 32)$$

$$\underline{5x + 0y = 40}$$

$$\frac{5x}{5} = \frac{40}{5}$$

$$\leftarrow \boxed{x = 8}$$

$$\frac{3}{4}(8) - y = 2$$

$$\frac{24}{4} - y = 2$$

$$6 - y = 2$$

$$\underline{-6 \quad -6}$$

$$\underline{-y = -4}$$

$$\underline{-1 \quad -1}$$

$$\boxed{y = 4}$$

verify: $\textcircled{1} \frac{3}{4}(8) - 4 = 2$

$$6 - 4 = 2$$

$$2 = 2 \checkmark$$

$$\textcircled{2} \frac{1}{8}(8) + \frac{1}{4}(4) = 2$$

$$1 + 1 = 2$$

$$2 = 2 \checkmark$$