

Lesson 7.4 ~ Using Substitution to Solve a System of Linear Equations

The second of three methods is substitution. We will solve for a variable in one equation, and then substitute it into the second equation.

Substitution Steps:

1. Solve for one variable from either equation (it is simplest to select the variable whose coefficient is one).
2. Substitute the result from step 1 into the other equation.
3. Solve for the remaining variable.
4. Substitute known value into either original equation and solve.
5. Verify the solution.

Example #1: Solve the linear system and verify the solution.

$$\textcircled{1} 3x + y = 3$$

$$\textcircled{2} 7x - 2y = 20$$

$$\textcircled{1} 3x + y = 3$$

$$-3x \quad -3x$$

$$y = -3x + 3 \rightarrow$$

$$\textcircled{2} 7x - 2(-3x + 3) = 20$$

$$7x + 6x - 6 = 20$$

$$+6 \quad +6$$

$$\frac{13x}{13} = \frac{26}{13}$$

$$\boxed{x = 2}$$

$$y = -3(2) + 3$$

$$y = -6 + 3$$

$$\boxed{y = -3}$$

verify: $\textcircled{1} 3(2) + (-3) = 3$

$$6 - 3 = 3$$

$$3 = 3 \checkmark$$

$\textcircled{2} 7(2) - 2(-3) = 20$

$$14 + 6 = 20$$

$$20 = 20 \checkmark$$

Example #2: Solve the linear system and verify the solution.

$$\textcircled{1} x - 6y = 4$$

$$\textcircled{2} -2x + y = -8$$

$$\textcircled{1} \begin{array}{r} x - 6y = 4 \\ +6y + 6y \end{array}$$

$$x = 6y + 4 \rightarrow \textcircled{2} \begin{array}{r} -2(6y + 4) + y = -8 \\ -12y - 8 + y = -8 \\ -11y = 0 \end{array}$$

$$x = 6(0) + 4 \quad \boxed{y = 0}$$

$$\boxed{x = 4}$$

verify: $\textcircled{1} 4 - 6(0) = 4$

$$4 = 4 \checkmark$$

$$\textcircled{2} -2(4) + 0 = -8$$

$$-8 = -8 \checkmark$$

Example #3: Solve the linear system and verify the solution.

$$\textcircled{1} \left(\frac{1}{2}x - \frac{4}{5}y = -2 \right) \times 10 \Rightarrow 5x - 8y = -20$$

$$\textcircled{2} \left(y = \frac{1}{4}x - \frac{3}{8} \right) \times 8 \Rightarrow 8y = 2x - 3$$

$$\textcircled{1} 5x - (2x - 3) = -20$$

$$5x - 2x + 3 = -20$$

$$\frac{3x}{3} = \frac{-23}{3}$$

$$\boxed{x = -\frac{23}{3}}$$

$$\textcircled{2} y = \frac{1}{4} \left(-\frac{23}{3} \right) - \frac{3}{8}$$

$$y = \frac{-23}{12} - \frac{3}{8}$$

$$y = \frac{-46}{24} - \frac{9}{24}$$

$$\boxed{y = -\frac{55}{24}}$$

verify: $\textcircled{1} \frac{1}{2} \left(-\frac{23}{3} \right) - \frac{4}{5} \left(-\frac{55}{24} \right) = -2 \checkmark$

$$\textcircled{2} -\frac{55}{24} = \frac{1}{4} \left(-\frac{23}{3} \right) - \frac{3}{8} \checkmark$$