KEY

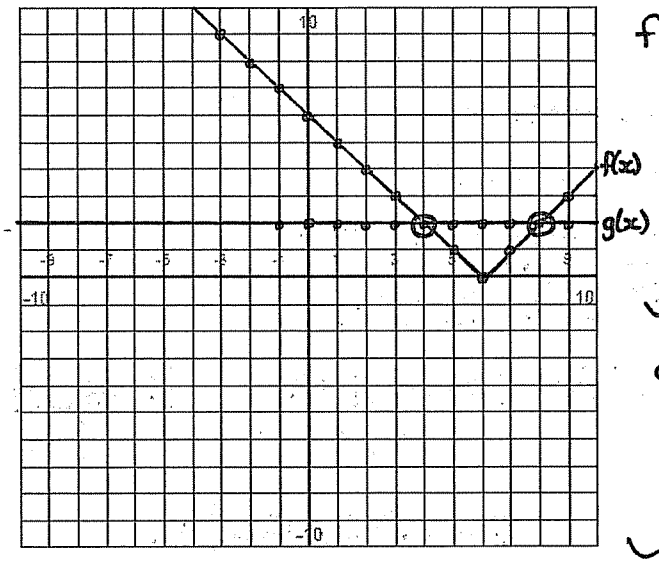
Pre-Calculus 11
Lesson 7.3 ~ Absolute Value Equations

An equation in which the variable occurs in an absolute value sign is called an absolute-value equation. We will use cases to solve these absolute-value equations.

- Case 1: the expression inside the absolute value symbol is positive or zero
- Case 2: the expression inside the absolute value symbol is negative

$y = mx$

Example #1: Solve $|6 - x| = 2$ graphically and algebraically.



$f(x) = |6-x|$ & $g(x) = 2$

or

$y = |6-x|$ & $y = 2$

$\therefore \boxed{x = 4, 8}$

case 1:

$6 - x = 2$

$\boxed{4 = x}$

case 2:

$6 - x = -2$

$\boxed{8 = x}$

verify: $|6-4| = 2$ $\quad |6-8| = 2$

$|2| = 2$ ✓ $\quad |-2| = 2$ ✓

Example #2: Solve $|x + 5| = 4x - 1$

$x + 5 = 4x - 1$

$\frac{6}{3} = \frac{3x}{3}$

$\boxed{2 = x}$

$x + 5 = -(4x - 1)$

$x + 5 = -4x + 1$

$5x = -4$

$x = -\frac{4}{5}$ ← extraneous root

verify: $|2+5| = 4(2) - 1$

$|7| = 7$ ✓

$|-\frac{4}{5} + 5| = 4(-\frac{4}{5}) - 1$

$|4.2| = -4.2$ ✗

Example #3: Solve $|4x-5|+9=2$ → $|4x-5| = 2-9$

$|4x-5| = -7$

↑ can never be negative, so this equation has no solution.

Example #4: Solve $|x^2-3x| = 2$

$x^2-3x = 2$ (−2, 1; −1, 2)

$x^2-3x-2=0$

↑ can't factor

$x = \frac{3 \pm \sqrt{3^2-4(-2)}}{2}$

$x = \frac{3 \pm \sqrt{17}}{2}$

$\boxed{x \doteq 3.562, -0.562}$

$x^2-3x = -2$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$\boxed{x=2}$ $\boxed{x=1}$

verify: $|2^2-3(2)| = 2$ $|1^2-3(1)| = 2$

$|-2| = 2$ ✓ $|-2| = 2$ ✓

$|3.562^2-3(3.562)| = 2$

$|2| = 2$ ✓

$|(-0.562)^2-3(-0.562)| = 2$

$|2| = 2$ ✓

Example #5: Solve $|x-5| = x^2-8x+15$

$x-5 = x^2-8x+15$

$0 = x^2-9x+20$

$0 = (x-4)(x-5)$

$x = 4, 5$

↑ extraneous root

verify:

$|4-5| = 4^2-8(4)+15$

$|-1| = -1$ ✗

$|5-5| = 5^2-8(5)+15$

$|0| = 0$ ✓

$x-5 = -(x^2-8x+15)$

$x-5 = -x^2+8x-15$

$x^2-7x+10=0$

$(x-2)(x-5)=0$

$\boxed{x=2, 5}$

$|2-5| = 2^2-8(2)+15$

$|-3| = 3$ ✓