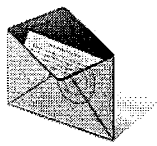
KEY

Foundations of Math & Pre-Calculus 10
Lesson 5.2 ~ Functions & Function Notation

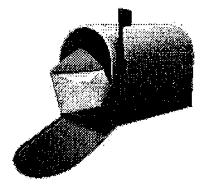
A **function** is a special type of relation where every input has one unique output.

The Mailbox Analogy:

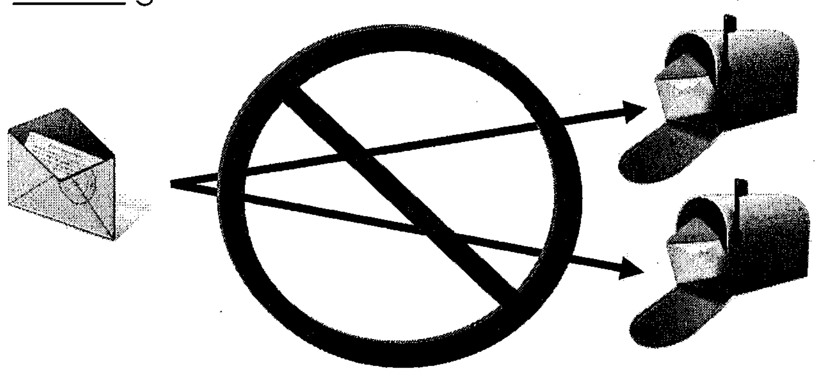
Think of the input as a letter.



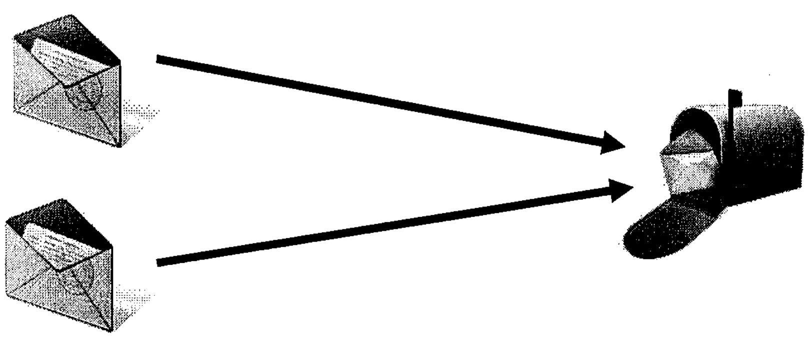
Think of the output as a mailbox.



The same letter *cannot* go to two different mailboxes: **THIS IS NOT A FUNCTION!**



Two different letters *can* go to the same mailbox: **THIS IS A FUNCTION!**



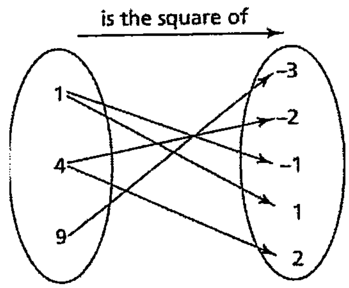
Example #1: Determine whether the relation is a function. Explain why or why not.

a) A relation that associates given shapes with the number of right angles in the shape. {(right triangle, 1), (square, 4), (rectangle, 4), (regular hexagon, 0)}

input	output
right Δ	1
square	4
rect.	4
reg. hex	0

★ This is a function because every input (shape) has only one output (number of right angles).

b)



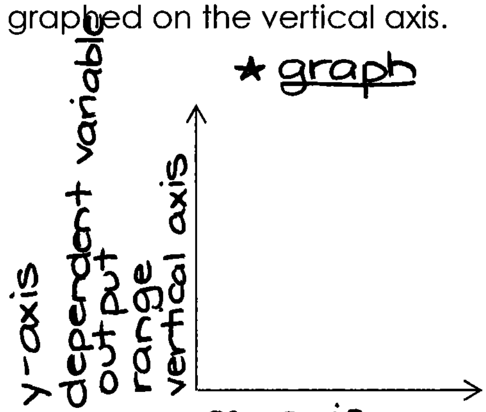
input	output
1	-1
1	1
4	-2
4	2
9	-3

★ This is not a function because some inputs have more than one output (input 1 has two outputs, -1 & 1).

Independent variables represent data that is not determined by the value of another variable (most common independent variable is time); graphed on the horizontal axis.

Dependent variables represent data that is determined by the value of another variable (common dependent variables are cost, height, distance, etc); graphed on the vertical axis.

★ graph



x-axis
independent variable
input
domain
horizontal axis

★ table of values

input	output
x	y
horiz.	vert.
domain	range
indep.	dep.

★ sentence

"The dependent variable is a function of the independent variable."

Example #2: The table shows the masses, m grams, of different numbers of identical marbles, n.

Number of Marbles, n	Mass of Marbles, m
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

a) Why is the relation also a function?

Every input (n) has one unique output (m).

b) Identify the independent variable and the dependent variable

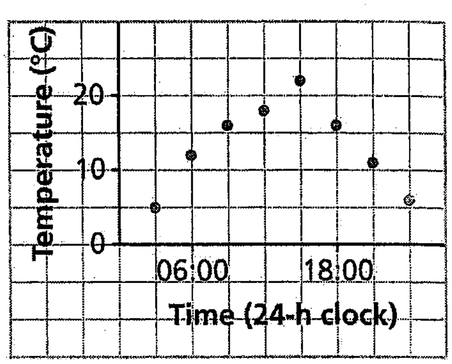
independent variable : n

dependent variable : m

Vertical Line Test: A graph represents a function when no two points on the graph lie on the same vertical line.

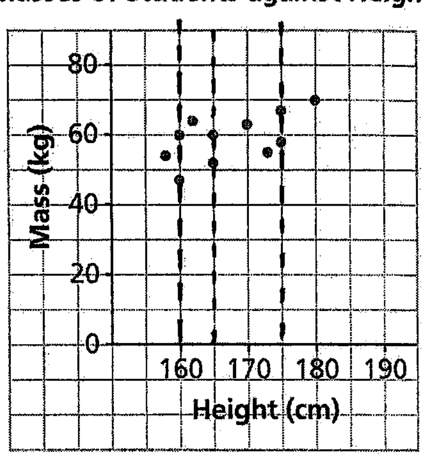
Example #3: Using the Vertical Line Test, determine whether each graph represents a function.

Outside Temperature over a 24-h Period



★ Function

Masses of Students against Height

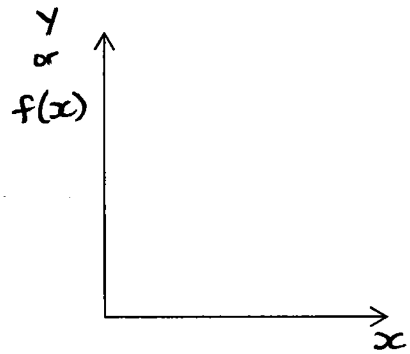


★ Not a function

Function Notation can be used to write equations in two variables.

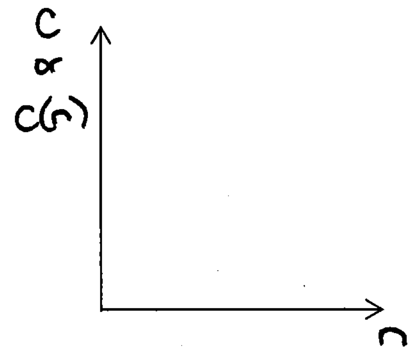
$y = -2x + 5$ can be written as $f(x) = -2x + 5$

- (say "f of x" → means f as a function of x)



$C = 40n + 5$ can be written as $C(n) = 40n + 5$, where C is the cost in dollars and n is the number of copies made

- (say "C of n" → means cost as a function of the number of copies)



Example #4: Find $g(5)$ for $g(x) = 3x + 1$

$g(5) = 3(5) + 1$

$g(5) = 15 + 1$

$\boxed{g(5) = 16}$

Example #5: Find x for $f(x) = 10$ and $f(x) = 4x - 2$

$10 = 4x \cancel{-2}$
$+2 \qquad \cancel{+2}$

$\frac{12}{4} = \frac{\cancel{4}x}{\cancel{4}}$

$3 = x \longrightarrow \boxed{f(3) = 10}$

Example #6: The equation $V = -0.08d + 50$ represents the volume, V litres, of gas remaining in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it is empty.

a) Describe the function and write it in function notation

The amount of gas in the vehicle's tank is a function of the number of kilometres driven.

$\boxed{V(d) = -0.08d + 50}$

b) Determine the value of $V(600)$. What does this number represent?

$V(600) = -0.08(600) + 50$

$V(600) = -48 + 50$

$\boxed{V(600) = 2}$ → After driving 600 km, there are 2 L of gas remaining.

c) Determine the value of d when $V(d) = 26$. What does this number represent?

$26 = -0.08d \cancel{+50}$
$-50 \qquad \cancel{-50}$

$\frac{-24}{-0.08} = \frac{-0.08d}{-0.08}$

$300 = d$ → $V(300) = 26$

→ After driving 300 km, there are 26 L of gas remaining.