A function is a special type of relation where every input has one unique output.

The Mailbox Analogy:
Think of the input as a letter. Think of the output as a mailbox.

The same letter cannot go to two different mailboxes: THIS IS NOT A FUNCTION!

Two different letters can go to the same mailbox: THIS IS A FUNCTION!
Example #1: Determine whether the relation is a function. Explain why or why not.

a) A relation that associates given shapes with the number of right angles in the shape. \{(\text{right triangle}, 1), (\text{square}, 4), (\text{rectangle}, 4), (\text{regular hexagon}, 0)\}

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>right Δ</td>
<td>1</td>
</tr>
<tr>
<td>square</td>
<td>4</td>
</tr>
<tr>
<td>rect.</td>
<td>4</td>
</tr>
<tr>
<td>reg. hex</td>
<td>0</td>
</tr>
</tbody>
</table>

* This is a function because every input (shape) has only one output (number of right angles).

b)

is the square of

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>-3</td>
</tr>
</tbody>
</table>

* This is not a function because some inputs have more than one output (input 1 has two outputs, -1 & 1).

**Independent** variables represent data that is not determined by the value of another variable (most common independent variable is time); graphed on the horizontal axis.

**Dependent** variables represent data that is determined by the value of another variable (common dependent variables are cost, height, distance, etc); graphed on the vertical axis.

**table of values**

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>horiz. vert. domain range indep. dep.</td>
<td></td>
</tr>
</tbody>
</table>

**sentence**

"The dependent variable is a function of the independent variable."
Example #2: The table shows the masses, $m$ grams, of different numbers of identical marbles, $n$.

<table>
<thead>
<tr>
<th>Number of Marbles, $n$</th>
<th>Mass of Marbles, $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.27</td>
</tr>
<tr>
<td>2</td>
<td>2.54</td>
</tr>
<tr>
<td>3</td>
<td>3.81</td>
</tr>
<tr>
<td>4</td>
<td>5.08</td>
</tr>
<tr>
<td>5</td>
<td>6.35</td>
</tr>
<tr>
<td>6</td>
<td>7.62</td>
</tr>
</tbody>
</table>

a) Why is the relation also a function?

Every input ($n$) has one unique output ($m$).

b) Identify the independent variable and the dependent variable

independent variable : $n$

dependent variable : $m$

Vertical Line Test: A graph represents a function when no two points on the graph lie on the same vertical line.

Example #3: Using the Vertical Line Test, determine whether each graph represents a function.

Outside Temperature over a 24-h Period

* Function

Masses of Students against Height

* Not a function
Function Notation can be used to write equations in two variables.

\[ y = -2x + 5 \text{ can be written as } f(x) = -2x + 5 \]

- (say "f of x" → means f as a function of x)

\[ C = 40n + 5 \text{ can be written as } C(n) = 40n + 5, \]

where \( C \) is the cost in dollars and \( n \) is the number of copies made

- (say "C of n" → means cost as a function of the number of copies)

Example #4: Find \( g(5) \) for \( g(x) = 3x + 1 \)

\[
\begin{align*}
g(5) &= 3(5) + 1 \\
g(5) &= 15 + 1 \\
g(5) &= 16
\end{align*}
\]

Example #5: Find \( x \) for \( f(x) = 10 \) and \( f(x) = 4x - 2 \)

\[
\begin{align*}
10 &= 4x - 2 \\
+2 &= 4x \\
\frac{12}{4} &= x \\
3 &= x
\end{align*}
\]

\[ f(3) = 10 \]

Example #6: The equation \( V = -0.08d + 50 \) represents the volume, \( V \) litres, of gas remaining in a vehicle’s tank after travelling \( d \) kilometres. The gas tank is not refilled until it is empty.

a) Describe the function and write it in function notation

The amount of gas in the vehicle’s tank is a function of the number of kilometres driven.

\[ V(d) = -0.08d + 50 \]

b) Determine the value of \( V(600) \). What does this number represent?

\[
\begin{align*}
V(600) &= -0.08(600) + 50 \\
&= -48 + 50 \\
&= 2
\end{align*}
\]

\[ \rightarrow \text{After driving 600 km, there are 2 L of gas remaining.} \]

c) Determine the value of \( d \) when \( V(d) = 26 \). What does this number represent?

\[
\begin{align*}
26 &= -0.08d + 50 \\
-50 &= -0.08d \\
\frac{-24}{-0.08} &= d \\
300 &= d
\end{align*}
\]

\[ \rightarrow V(300) = 26 \]

\[ \rightarrow \text{After driving 300 km, there are 26 L of gas remaining.} \]