Pre-Calculus 11
Lesson 4.4 ~ The Quadratic Formula

We have learned how to solve quadratic equations of the form $ax^2+bx+c=0$ by graphing and factoring. One final method that can be used to solve quadratic equations is the **quadratic formula**. While the other methods have some limitations, the quadratic formula can be used to solve *any* quadratic equation.

Remember: when we are asked to **solve** a quadratic equation, we are looking for the possible values of x that will satisfy the equation. In other words, we want to find what values of x will make our equation equal zero.

Quadratic Formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ for $ax^2+bx+c=0$, where $a\neq 0$.

To solve the quadratic equation $x^2-6x+7=0$, the first step we could take is to factor the equation...but in this case we cannot find numbers that have a product of 7 and a sum of -6.

If the equation cannot be factored, we can substitute the values of a, b, and c into the quadratic formula.

$$\left.\begin{aligned} a&=1\\ b&=-6\\ c&=7 \end{aligned}\right\} \quad x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(7)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{36-28}}{2}$$

$$x=\frac{6\pm\sqrt{8}}{2}$$

$$x=\frac{6\pm2\sqrt{2}}{2}$$

$$x=3\pm\sqrt{2}$$

$\boxed{x=3+\sqrt{2} \text{ and } x=3-\sqrt{2}}$

Example #1: Determine the roots of the quadratic equation $x^2 - 5x - 7 = 0$.

$$\left.\begin{aligned} a &= 1 \\ b &= -5 \\ c &= -7 \end{aligned}\right\}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 + 28}}{2}$$

$$x = \frac{5 \pm \sqrt{53}}{2}$$

$$x_1 = \frac{5 + \sqrt{53}}{2} \doteq \boxed{6.14} \qquad x_2 = \frac{5 - \sqrt{53}}{2} \doteq \boxed{-1.14}$$

Example #2: Find the zeroes of the quadratic function $f(x) = 12x^2 - 35x + 18$.

$$\left.\begin{aligned} a &= 12 \\ b &= -35 \\ c &= 18 \end{aligned}\right\}$$

$$x = \frac{-(-35) \pm \sqrt{(-35)^2 - 4(12)(18)}}{2(12)}$$

$$x = \frac{35 \pm \sqrt{361}}{24}$$

$$x = \frac{35 + 19}{24}$$

$$x_1 = \frac{54}{24} = \frac{27}{12} = \boxed{2.25} \qquad x_2 = \frac{16}{24} = \frac{4}{6} = \frac{2}{3} \doteq \boxed{0.\bar{6}}$$

When we first began graphing quadratic functions, we noticed that while most parabolas intersect the x-axis twice, some have their vertex on the x-axis and other do not intersect the x-axis at all. We will now use a part of the quadratic formula to determine the number of roots of a quadratic equation, known as the **nature of the roots.**

Determine the number of roots for the following quadratic equations.

a) $x^2 - 6x + 5 = 0$

$$x = \frac{6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{16}}{2} \Longleftarrow \text{Positive}$$

$$x = \frac{6 \pm 4}{2}$$

$$\boxed{x = 5 \text{ and } x = 1} \Longleftarrow \text{Two Different Roots}$$

b) $x^2 - 6x + 9 = 0$

$$x = \frac{6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{0}}{2} \Longleftarrow \text{Zero}$$

$$x = \frac{6 \pm 0}{2}$$

$$\boxed{x = 3 \text{ and } x = 3} \Longleftarrow \text{Two Equal Roots}$$

OR One Root

c) $x^2 - 6x + 13 = 0$

$$x = \frac{6 \pm \sqrt{6^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-16}}{2} \Longleftarrow \text{Negative - No Roots}$$

We can summarize the above findings:

- If $b^2 - 4ac > 0$, there are two different real roots
- If $b^2 - 4ac = 0$, there are two equal real roots
- If $b^2 - 4ac < 0$, there are no real roots.

Discriminant: $b^2 - 4ac$ is the discriminant of the equation $ax^2 + bx + c = 0$ because it discriminates among the three cases that can occur.

Example #3: Without solving each equation, determine the nature of the roots.

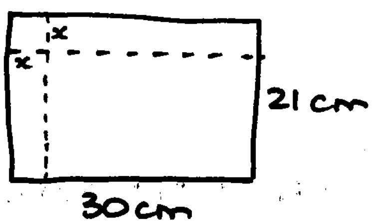
a) $4x^2 - 12x + 9 = 0$

$$b^2 - 4ac = (-12)^2 - 4(4)(9)$$
$$= 144 - 144$$
$$= 0 \rightarrow \boxed{\text{one solution}}$$

b) $x^2 - 2x + 3 = 0$

$$b^2 - 4ac = (-2)^2 - 4(1)(3)$$
$$= 4 - 12$$
$$= -8 \rightarrow \boxed{\text{no solution}}$$

Example #4: A picture measures 30 cm by 21 cm. You crop the picture by removing strips of the same width from the top and one side of the picture. This reduces the area to 40% of the original area. Determine the width of the removed strips.



$$30 \times 21 = 630 \text{ cm}^2$$
$$630 \times 0.4 = 252 \text{ cm}^2$$

$$252 = (30 - x)(21 - x)$$
$$252 = 630 - 30x - 21x + x^2$$
$$252 = 630 - 51x + x^2$$
$$0 = x^2 - 51x + 378$$

$$x = \frac{-(-51) \pm \sqrt{(-51)^2 - 4(1)(378)}}{2(1)}$$

$$x = \frac{51 \pm \sqrt{1089}}{2} = \frac{51 \pm 33}{2} = 42, 9$$

→ 42 is too big to cut away, so

$\boxed{x = 9 \text{ cm}}$