KEY

Pre-Calculus 11
Lesson 4.3 ~ Solving Quadratic Equations by Completing the Square

The second algebraic method we will learn for solving quadratic equations is by completing the square. Once an equation is in vertex form, you can use algebra to isolate the variable.

Example #1: Determine the root(s) of the quadratic equation $x^2 = 9$. Verify your answer(s).

$\sqrt{x^2} = \sqrt{9}$

$\boxed{x = \pm 3}$

verify: $3^2 = 9$?

$9 = 9$ ✓

$(-3)^2 = 9$?

$9 = 9$ ✓

Example #2: Solve the quadratic equation $3(x + 4)^2 = 12$. Verify your answer(s).

$\frac{3(x+4)^2}{3} = \frac{12}{3}$

$\sqrt{(x+4)^2} = \sqrt{4}$

$x + 4 = \pm 2$

$-4 \quad -4$

$x = 2 - 4 \qquad x = -2 - 4$

$\boxed{x = -2} \qquad \boxed{x = -6}$

verify: $3(-2+4)^2 = 12$?

$3(2)^2 = 12$?

$12 = 12$ ✓

$3(-6+4)^2 = 12$?

$3(-2)^2 = 12$?

$12 = 12$ ✓

Example #3: Find the x-intercept(s) of the graph of $-2(x - 3)^2 - 1 = 0$ to the nearest hundredth. Verify your answer(s).

$-2(x-3)^2 - 1 = 0$

$+1 \quad +1$

$\frac{-2(x-3)^2}{-2} = \frac{1}{-2}$

$\sqrt{(x-3)^2} = \sqrt{-1/2}$

cannot square root a negative number, so this quadratic has no solution (no x-intercepts, no zeros, no roots)

Example #4: A defender kicks a soccer ball away from her own goal. The path of the kicked soccer ball can be approximated by the quadratic function $h(x) = -0.016(x - 36)^2 + 5.536$, where x is the horizontal distance travelled, in metres, from the goal line and h is the height, in metres.

a) How far, to the nearest tenth of a metre, is the soccer ball from the goal line when it is kicked?

$h = 0$, so $0 = -0.016(x-36)^2 + 5.536$

$-5.536 \quad -5.536$

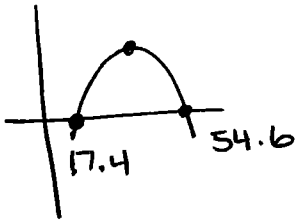
$-5.536 = -0.016(x-36)^2$

$\frac{-5.536}{-0.016} = \frac{-0.016(x-36)^2}{-0.016}$

$\pm\sqrt{346} = \sqrt{(x-36)^2}$

$+36 \quad +36$

$36 \pm \sqrt{346} = x \doteq 54.6, 17.4$



→ The ball is $\boxed{17.4\text{m}}$ from the goal line when it is kicked.

b) How far, to the nearest tenth of a metre, does the soccer ball travel before it hits the ground?

$54.6 - 17.4 = 37.2$

→ The ball travels $\boxed{37.2\text{m}.}$