

Pre-Calculus 11  
Lesson 4.2 ~ Factoring Quadratic Equations

The first algebraic method we will learn for solving quadratic equations is by factoring. Before we begin solving equations, let's practice factoring expressions.

Example #1: Factor the following expressions.

a)  $3x^2 + 3x - 6$

$$3(x^2 + x - 2)$$

$$\boxed{3(x+2)(x-1)}$$

b)  $0.49r^2 - 36t^2$

$$\boxed{(0.7r + 6t)(0.7r - 6t)}$$

c)  $-2(n+3)^2 + 12(n+3) + 14$

let  $k = n+3$

then  $-2k^2 + 12k + 14$

$$-2(k^2 - 6k - 7)$$

$$-2(k+1)(k-7)$$

and  $-2(n+3+1)(n+3-7)$

$$\boxed{-2(n+4)(n-4)}$$

d)  $4(x-2)^2 - 0.25(y-4)^2$

let  $a = x-2$  &  $b = y-4$

then  $4a^2 - 0.25b^2$

$$(2a + 0.5b)(2a - 0.5b)$$

and  $(2(x-2) + 0.5(y-4))(2(x-2) - 0.5(y-4))$

$$(2x-4+0.5y-2)(2x-4-0.5y+2)$$

$$\boxed{(2x+0.5y-6)(2x-0.5y-2)}$$

The Zero Product Property states that for any real numbers  $a$  and  $b$ , if  $ab=0$ , then  $a=0$ ,  $b=0$ , or both  $a=b=0$ .

Solve the quadratic equation  $x^2 + 7x + 12 = 0$  by factoring.

$$x^2 + 7x + 12 = 0$$

$$(x+3)(x+4) = 0$$

$$(x+3) = 0 \quad \text{or} \quad (x+4) = 0$$

$$x = -3 \quad \quad \quad x = -4$$

\*\*Check:  $(-3)^2 + 7(-3) + 12 = 0?$    $(-4)^2 + 7(-4) + 12 = 0?$

$$9 - 21 + 12 = 0? \quad \quad \quad 16 - 28 + 12 = 0?$$

Example #2: Solve the quadratic equation  $x^2 - 4x - 32 = 0$

$$(x - 8)(x + 4) = 0$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ x - \cancel{8} = 0 & \text{or} & x + \cancel{4} = 0 \\ \quad \quad \quad +8 \quad +8 & & \quad \quad \quad -4 \quad -4 \\ \boxed{x = 8} & & \boxed{x = -4} \end{array}$$

verify:  $8^2 - 4(8) - 32 = 0?$   
 $64 - 32 - 32 = 0?$   
 $0 = 0 \checkmark$

$(-4)^2 - 4(-4) - 32 = 0?$   
 $16 + 16 - 32 = 0?$   
 $0 = 0 \checkmark$

Example #3: Solve the quadratic equation  $3x^2 + 5x = 0$  by factoring.

$$x(3x + 5) = 0$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \boxed{x = 0} & \text{or} & 3x + \cancel{5} = 0 \\ & & \quad \quad \quad -\cancel{5} \quad -5 \\ & & \frac{3x}{3} = \frac{-5}{3} \\ & & \boxed{x = -5/3} \end{array}$$

verify:  $3(0)^2 + 5(0) = 0?$   
 $0 + 0 = 0?$   
 $0 = 0 \checkmark$

$3(-5/3)^2 + 5(-5/3) = 0?$   
 $\cancel{3}(\frac{25}{9}) + -25/3 = 0?$   
 $25/3 + -25/3 = 0?$   
 $0 = 0 \checkmark$

Example #4: Solve the quadratic equation  $2x^2 + 3 = 5x + 1$  by factoring.

$$2x^2 + 3 = 5x + 1$$

$$-5x - 1 \quad -5x - 1$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$2x(x-2) - 1(x-2) = 0$$

$$(2x-1)(x-2) = 0$$

$$\downarrow \qquad \qquad \downarrow$$

$$2x-1=0 \qquad x-2=0$$

$$2x=1 \qquad \boxed{x=2}$$

$$\boxed{x=1/2}$$

verify:  $2(1/2)^2 + 3 = 5(1/2) + 1?$

$$2(1/4) + 3 = 5/2 + 1?$$

$$2/4 + 3 = 5/2 + 1?$$

$$1/2 + 6/2 = 5/2 + 2/2?$$

$$7/2 = 7/2 \checkmark$$

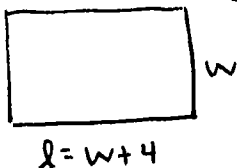
$$2(2)^2 + 3 = 5(2) + 1?$$

$$8 + 3 = 10 + 1?$$

$$11 = 11 \checkmark$$

Example #5: The area of a rectangular Ping Pong table is 45 ft<sup>2</sup>. The length is 4 ft more than the width. What are the dimensions of the table?

Let  $l$  = length,  $w$  = width, &  $A$  = area. Then



$$A = 45 = lw$$

$$45 = (w+4)(w)$$

$$45 = w^2 + 4w$$

$$0 = w^2 + 4w - 45$$

$$0 = (w+9)(w-5)$$

$$\downarrow$$

$$w = -9$$

$\downarrow$   
can't be  
a negative  
length

$$\downarrow$$

$$w = 5$$

$$\downarrow$$

width =  $\boxed{5 \text{ ft}}$

length =  $5 + 4 = \boxed{9 \text{ ft}}$

area =  $5 \times 9 = 45 \checkmark$