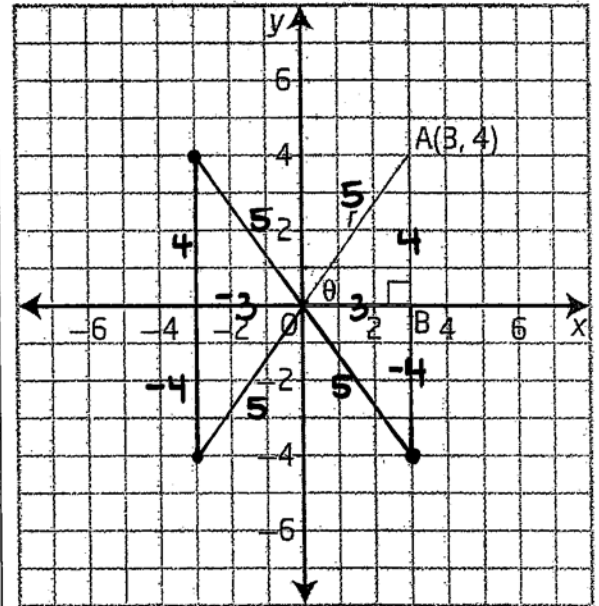


Lesson 2.2 ~ Trigonometric Ratios of Any Angle

Trigonometric Ratios for Angles Greater Than 90°

Quad	Sine	Cosine	Tangent
I	$\sin \theta = \frac{4}{5}$	$\cos \theta = \frac{3}{5}$	$\tan \theta = \frac{4}{3}$
II	$\sin \theta = \frac{4}{5}$	$\cos \theta = \frac{-3}{5}$	$\tan \theta = \frac{4}{-3}$
III	$\sin \theta = \frac{-4}{5}$	$\cos \theta = \frac{-3}{5}$	$\tan \theta = \frac{-4}{-3}$ $= \frac{4}{3}$
IV	$\sin \theta = \frac{-4}{5}$	$\cos \theta = \frac{3}{5}$	$\tan \theta = \frac{-4}{3}$



$$3^2 + 4^2 = r^2$$

$$25 = r^2$$

$$5 = r$$

Finding the Trigonometric Ratios of Any Angle  $\theta$ , where  $0^\circ \leq \theta < 360^\circ$  (CAST Rule)

Sin is (+) | All are (+)  
(cos/tan (-))

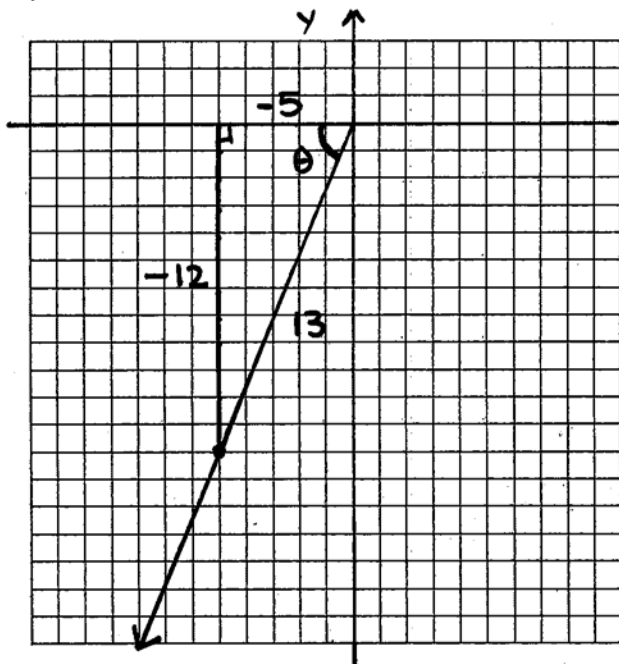
S	A
T	C

or

Students	All
Take	Calculus

Tan is (+) | Cos is (+)  
(sin/cos (-)) | (sin/tan (-))

Example #1: The point  $P(-5, -12)$  lies on the terminal arm of an angle,  $\theta$ , in standard position. Determine the exact trigonometric ratios for  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$ .



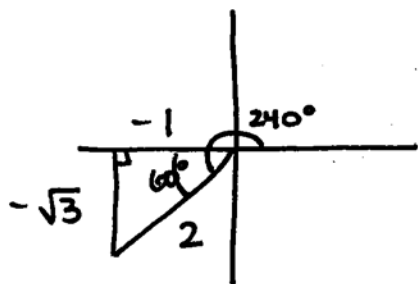
$$\begin{aligned} 5^2 + 12^2 &= r^2 \\ 25 + 144 &= r^2 \\ 169 &= r^2 \\ 13 &= r \end{aligned}$$

$$\sin \theta = \boxed{\frac{-12}{13}}$$

$$\cos \theta = \boxed{\frac{-5}{13}}$$

$$\tan \theta = \frac{-12}{-5} = \boxed{\frac{12}{5}}$$

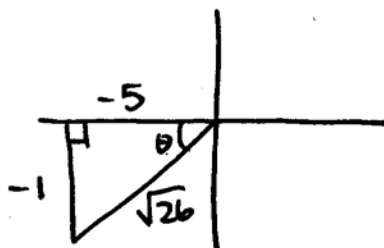
Example #2: Determine the exact value of  $\sin 240^\circ$ .



$$240 - 180 = 60^\circ \leftarrow \text{reference angle}$$

$$\sin 240^\circ = \boxed{\frac{-\sqrt{3}}{2}}$$

Example #3: Suppose  $\theta$  is an angle in standard position with terminal arm in quadrant III, and  $\tan \theta = \frac{1}{5}$ . Determine the exact values of  $\sin \theta$  and  $\cos \theta$ .



$$\tan \theta = \frac{1}{5} = \frac{\text{opp}}{\text{adj}}$$

$$1^2 + 5^2 = r^2$$

$$1 + 25 = r^2$$

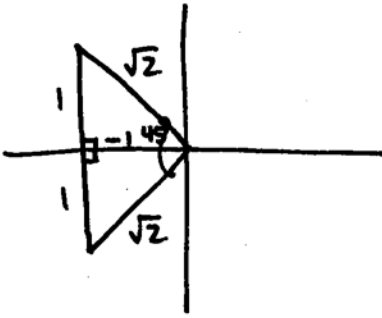
$$26 = r^2$$

$$\sqrt{26} = r$$

$$\sin \theta = \frac{-1}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}} = \boxed{\frac{-\sqrt{26}}{26}}$$

$$\cos \theta = \frac{-5}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}} = \boxed{\frac{-5\sqrt{26}}{26}}$$

Example #4: Solve  $\cos \theta = -\frac{1}{\sqrt{2}}$ ,  $0^\circ \leq \theta < 360^\circ$ .



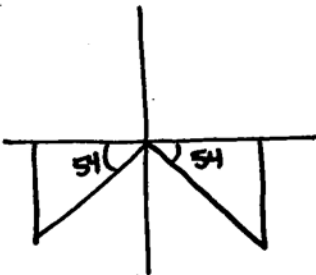
$$\cos \theta = -\frac{1}{\sqrt{2}} = \frac{\text{adj}}{\text{hyp}}$$

cos is (-) in quad II & III

$$\theta_1 = 180 - 45 = \boxed{135^\circ}$$

$$\theta_2 = 180 + 45 = \boxed{225^\circ}$$

Example #5: Determine the measure of  $\theta$ , to the nearest degree, given  $\sin \theta = -0.8090$ , where  $0^\circ \leq \theta < 360^\circ$ .



$$\sin^{-1}(-0.8090) = \theta = -54^\circ$$

sin is (-) in quad III & IV

$$\theta_1 = 180 + 54 = \boxed{234^\circ}$$

$$\theta_2 = 360 - 54 = \boxed{306^\circ}$$