KEY

Chapter 7 Review

7.1 Absolute Value, pages 278–284

1. Evaluate.
   a) $|-17| =$ 17
   b) $-\left|-1\frac{1}{2}\right| =$ $-1\frac{1}{2}$
   c) $|1.02| =$ 1.02

2. Arrange the numbers in order from greatest to least.
   $|-20.1|, -|20|, \left|\frac{41}{2}\right|, -20.2, \left|-19\frac{3}{4}\right|, -19.65$

   $-20.2, -|20|, -19.65, \left|-19\frac{3}{4}\right|, |-20.1|, \left|\frac{41}{2}\right|$

3. Evaluate.
   a) $|0 - 18| = |-18| = \boxed{18}$
   b) $-2|10.5| + |(-3)^3| = -\cancel{20}\ 21 + |-27| = -21 + 27 = \boxed{6}$
   c) $|-20 + 3(-2)^2| = |-20 + 12| = |-8| = \boxed{8}$

4. Insert absolute value symbols to make each statement true.
   a) $|9 - 12| + (-2)(4) = -5$
   b) $|(1.3 - 3.3)^3| = 8$
   c) $8 - 11|(12 - 15)| = -25$

5. The deepest point in Lake Superior is 732 feet below sea level.
   The surface elevation is 600 feet above sea level.
   a) Write an absolute value statement to determine the maximum depth.

   $|600 - (-732)| = d$, where d = max depth.

   +600

   0

   −732

   b) What is the depth?

   $d = \boxed{1332\text{ m}}$

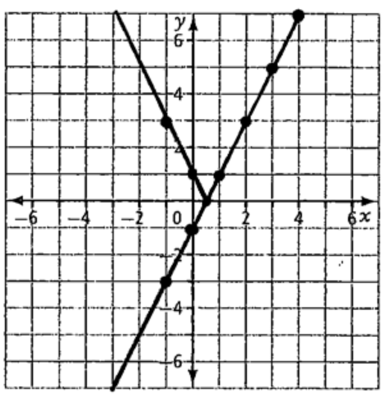
7.2 Absolute Value Functions, pages 285–296

6. Consider the functions $f(x) = 2x - 1$ and $g(x) = |2x - 1|$.

a) Complete the table.

x	$f(x)$	$g(x)$
−1	−3	3
0	−1	1
1	1	1
2	3	3
3	5	5
4	7	7

b) Sketch the graphs of $f(x)$ and $g(x)$.



c) Complete the table.

Characteristic	$f(x)$	$g(x)$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y \in \mathbb{R}$	$y \geq 0, y \in \mathbb{R}$
x-intercepts	$x = \frac{1}{2}$	$x = \frac{1}{2}$
y-intercepts	−1	1
Piecewise function	N/A	

$\rightarrow y = \begin{cases} 2x-1 & \text{for } x \geq \frac{1}{2} \\ -2x+1 & \text{for } x < \frac{1}{2} \end{cases}$

7. Consider the functions $f(x) = -x^2 + 4$ and $g(x) = |-x^2 + 4|$.

a) Complete the table.

x	$f(x)$	$g(x)$
−3	−5	5
−2	0	0
−1	3	3
0	4	4
1	3	3
2	0	0
3	−5	5

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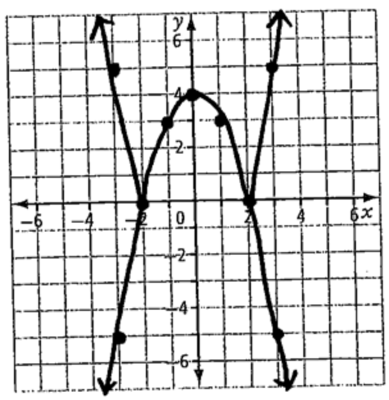
b) Sketch the graphs of $f(x)$ and $g(x)$.



c) Complete the table.

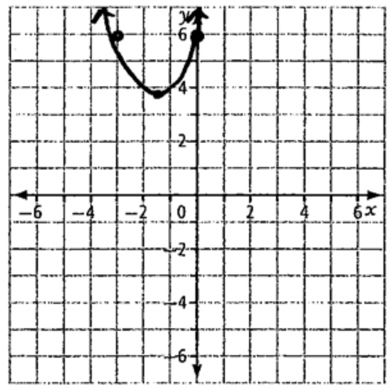
Characteristic	$f(x)$	$g(x)$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y \leq 4$	$y \geq 0$
x-intercepts	$x = \pm 2$	$x = \pm 2$
y-intercepts	$y = 4$	$y = 4$
Similarities	x-intercepts & y-intercept	
Differences	shape, range	
Piecewise function	N/A	

$$y = \begin{cases} -x^2+4 & \text{for } -2 \leq x \leq 2 \\ x^2-4 & \text{for } x < -2 \text{ \& } x > 2 \end{cases}$$

8. a) Without graphing, predict how the graphs of $f(x) = x^2 + 3x + 6$ and $g(x) = |x^2 + 3x + 6|$ are related.

f(x) is a parabola above the x-axis, so all y-values are already positive; ∴ f(x) = g(x) when graphed

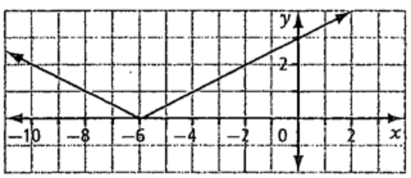
b) Check your prediction by using technology. Sketch the graph(s) below.



$x^2 + 3x + 2.25 - 2.25 + 6$

$(x+1.5)^2 + 3.75$

9. Write an absolute value function of the form $y = |ax + b|$ that has the following graph.



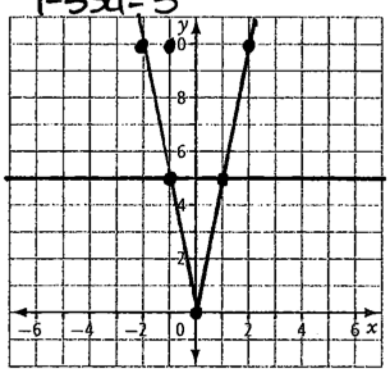
$y = \left|\frac{1}{2}x + 3\right|$

7.3 Absolute Value Equations, pages 297–308

10. Solve each absolute value equation by graphing.

a) $|-5x| + 4 = 9$

$|-5x| = 5$



$x = -1, 1$

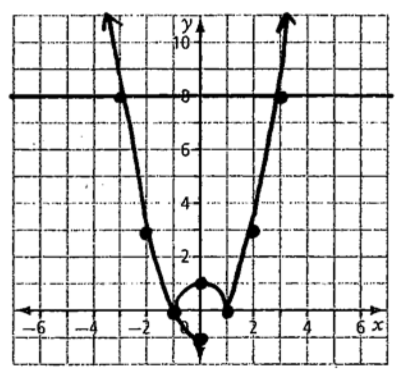
verify:

$|-5(-1)| + 4 = 9$ $\quad |-5(1)| + 4 = 9$

$|5| + 4 = 9$ $\quad |-5| + 4 = 9$

$9 = 9$ ✓ $\quad 9 = 9$ ✓

b) $|x^2 - 1| = 8$



$x = -3, 3$

verify: $|(-3)^2 - 1| = 8$ $\quad |3^2 - 1| = 8$

$|9 - 1| = 8$ $\quad |9 - 1| = 8$

$8 = 8$ ✓ $\quad 8 = 8$ ✓

11. Solve each absolute value equation algebraically.

a) $|2x + 10| = 14$

$2x+10=14$
$2x=+4$
$\boxed{x=+2}$

$2x+10=-14$
$2x=-24$
$\boxed{x=-12}$

$|2(+2)+10|=14$
$|14|=14$ ✓

$|2(-12)+10|=14$
$|-24+10|=14$
$14=14$ ✓

b) $|3x - 6| = x + 4$

$3x-6=x+4$
$2x=10$
$\boxed{x=5}$

$3x-6=-x-4$
$4x=2$
$\boxed{x=\frac{1}{2}}$

$|3(5)-6|=5+4$
$|9|=9$ ✓

$|3(\frac{1}{2})-6|=\frac{1}{2}+4$
$|-4.5|=4.5$ ✓

c) $|x^2 - 8x + 12| = 20$

$x^2-8x+12=20$
$x^2-8x-8=0.$
$x=\dfrac{8\pm\sqrt{8^2+4(+8)}}{2}$
$x \doteq 8.9$

$x^2-8x+12=-20$
$x^2-8x+32=0.$
$8\pm\sqrt{8^2-4(32)}$
$x=$ no sol'n

12. The adult dose of a medicine is 50 mg, but it is acceptable for the dose to vary by 0.5 mg.

a) Write an absolute value equation to determine the limits of an acceptable dosage.

Let d = acceptable dosage

Then $|d-50|=0.5$

b) What are the limits?

$d-50=0.5$
$d=50.5$ mg

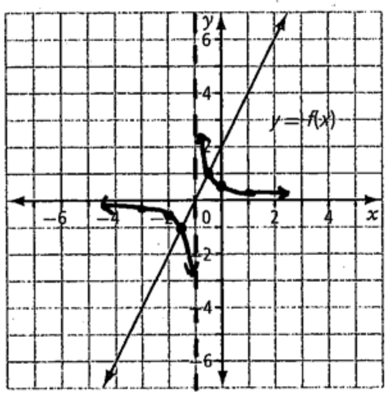
$d-50=-0.5$
$d=49.5$ mg

7.4 Reciprocal Functions, pages 309–323

13. For each graph of $y = f(x)$ below,

- Sketch the graph of the reciprocal function, $y = \frac{1}{f(x)}$, on the same grid.
- Find the asymptotes, invariant points, and intercepts.

a)



$y = 2x + 2$

$2x + 2 = 1$
$2x = -1$
$x = -\frac{1}{2}$
$x = -0.5$

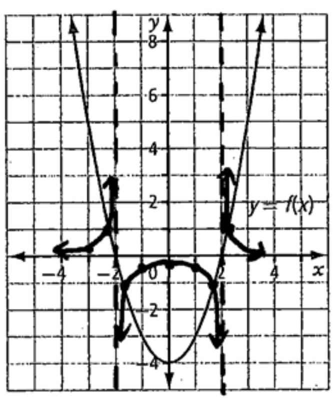
$2x + 2 = -1$
$2x = -3$
$x = -\frac{3}{2}$
$x = -1.5$

Asymptotes: $x = -1$

Invariant points: $(-1.5, -1)$, $(-0.5, 1)$

x-intercept: $\emptyset$; y-intercept: $(0, \frac{1}{2})$

b)



$y = x^2 - 4$

$x^2 - 4 = 1$
$x^2 = 5$
$x = \pm\sqrt{5}$
$x \approx \pm 2.2$

$x^2 - 4 = -1$
$x^2 = 3$
$x = \pm\sqrt{3}$
$x \approx 1.7$

Asymptotes: $x = -2$, $x = 2$

Invariant points: $(-2.2, 1)$, $(-1.7, -1)$, $(1.7, -1)$, $(2.2, 1)$

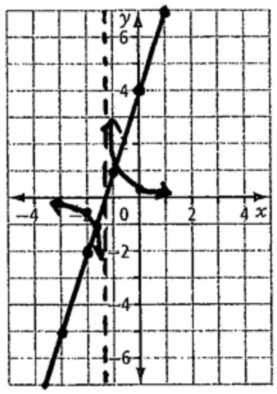
x-intercept: $\emptyset$; y-intercept: $(0, -\frac{1}{4})$

14. Sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes. Then, complete the table.

a) $f(x) = 3x + 4$

$3x+4=1$
$3x=-3$
$x=-1$

$3x+4=-1$
$3x=-5$
$x=-5/3$
$x=-1.\bar{6}$

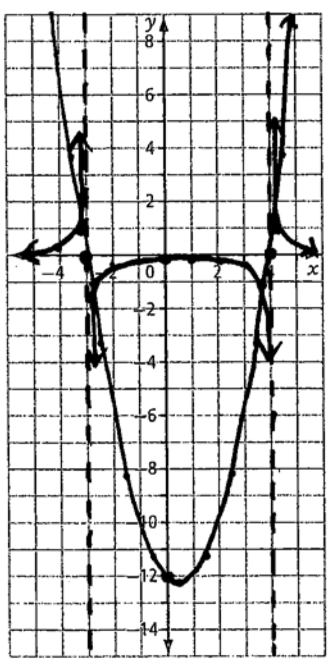


Characteristic	$f(x)$	$\frac{1}{f(x)}$
Asymptotes	N/A	$x=-1.\bar{3}$
x-intercept	$(-1.\bar{3},0)$	$\varnothing$
y-intercept	$(0,4)$	$(0,\frac{1}{4})$
Invariant points	$(-1.\bar{6},-1)$ &	$(-1,1)$
Domain	$x \in \mathbb{R}$	$x \neq -1.\bar{3}$
Range	$y \in \mathbb{R}$	$y \neq 0$

b) $f(x) = x^2 - x - 12$

$x^2-x-12=1$
$x^2-x-13=0$
$x=\frac{1 \pm \sqrt{1^2-4(1)(-13)}}{2(1)}$
$x=\frac{1 \pm \sqrt{53}}{2}$
$x \approx 4.1, -3.1$

$x^2-x-12=-1$
$x^2-x-11=0$
$x=\frac{1 \pm \sqrt{1^2-4(1)(11)}}{2(1)}$
$x=\frac{1 \pm \sqrt{45}}{2}$
$x \approx 3.9, -2.9$



Characteristic	$f(x)$	$\frac{1}{f(x)}$
Asymptotes	N/A	$x=-3$ $x=4$
x-intercept	$(-3,0)$ $(4,0)$	$\varnothing$
y-intercept	$(0,-12)$	$(0,-\frac{1}{12})$
Invariant points	$(-3.1,1)$ $(-2.9,-1)$	$(3.9,-1)$ $(4.1,1)$
Domain	$x \in \mathbb{R}$	$x \neq -3, 4$
Range	$y \geq -12.25$	$y \neq 0$

x^2-x-12
$(x-4)(x+3)$
$x=4,-3$

x^2-x-12
$x^2-x+\frac{1}{4}-\frac{1}{4}-12$
$(x-\frac{1}{2})^2-12.25$

15. The resistance, R, in ohms in an electric circuit is equal to the power, P, in watts, multiplied by the reciprocal of the square of the current, I, in amperes.

a) Write an equation to represent the relationship.

$$R = P \cdot \frac{1}{I^2}$$

b) What is the resistance, in ohms, for a circuit that uses 500 watts of power with a current of 2 amperes?

$$R = 500 \cdot \frac{1}{2^2}$$

$$R = \frac{500}{1} \cdot \frac{1}{4} = \frac{500}{4}$$

$R = 125$ ohms

c) The resistance in a circuit is 4 ohms. The same circuit uses 100 watts of power. Find the current in the circuit, in amperes.

$$4 = 100 \cdot \frac{1}{I^2}$$

$$\frac{4}{100} \times \frac{1}{I^2}$$

$$4I^2 = 100$$

$$\sqrt{I^2} = \sqrt{25}$$

$I = 5$ amperes

d) Find the power, in watts, when the current is 0.5 amperes and the resistance is 600 ohms.

$$600 = P \cdot \frac{1}{0.5^2}$$

$$600 = P \cdot \frac{1}{0.25} = P \cdot 4$$

$150 = P$
watts