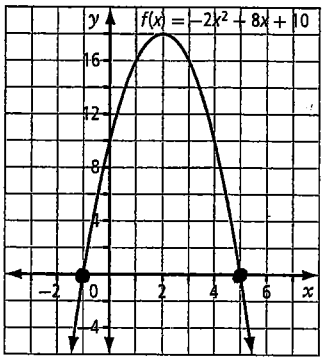
KEY

Chapter 4 Review

4.1 Graphical Solutions of Quadratic Equations, pages 147–155

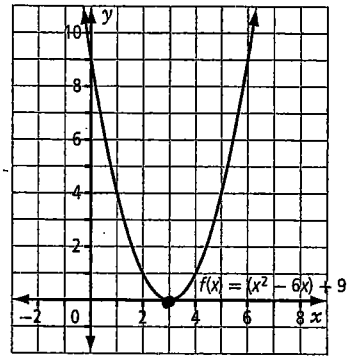
1. Use the graph to state the roots of each equation.

a)



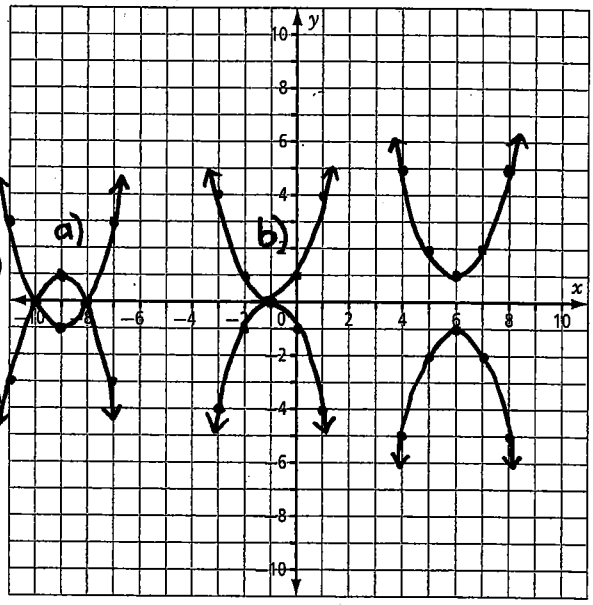
$x = -1, 5$

b)



$x = 3$

2. Explain which properties dictate the number of x-intercepts for each of the following. Then, sketch a sample of each type of graph on the same set of axes.



a) two distinct real roots

- vertex below x-axis & opening up ($q<0$ & $a>0$)
- vertex above x-axis & opening down ($q>0$ & $a<0$)

b) one real root

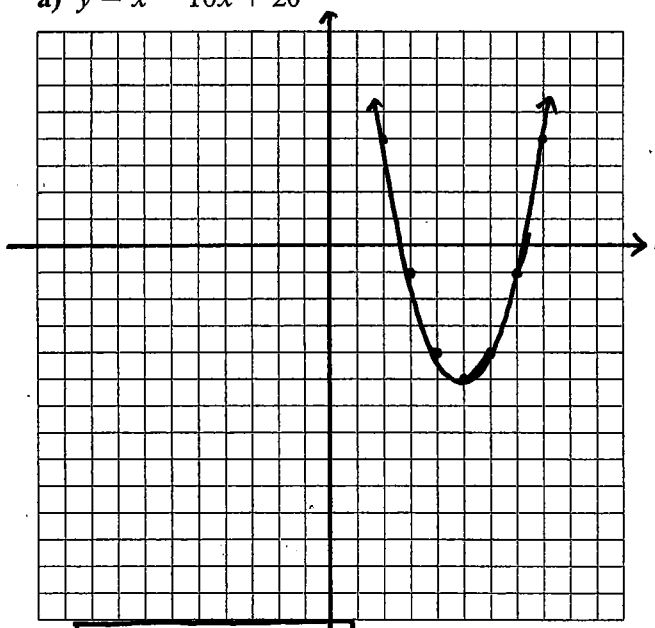
- vertex on x-axis (opening up or down) ($q=0$)

c) no real roots

- vertex below x-axis & opening down ($q<0$ & $a<0$)
- vertex above x-axis & opening up ($q>0$ & $a>0$)

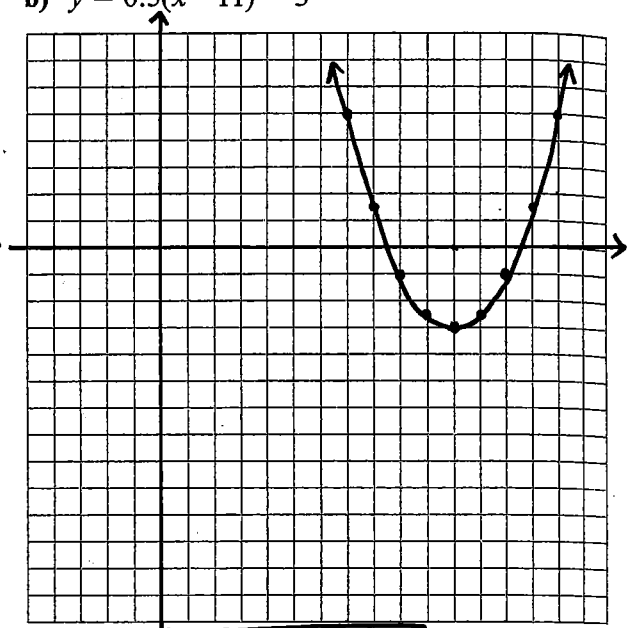
3. Graph the following. From your graph, state the roots to the nearest tenth.

a) $y = x^2 - 10x + 20$

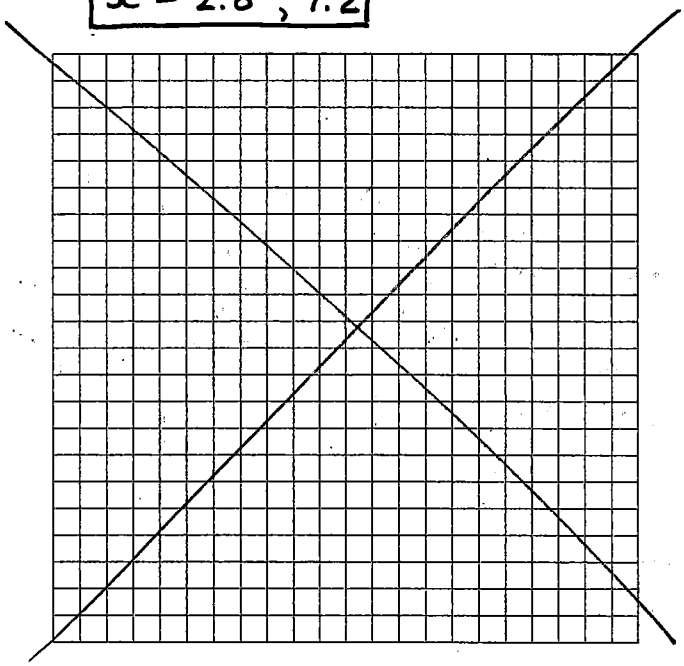


$x \doteq 2.8, 7.2$

b) $y = 0.5(x - 11)^2 - 3$



$x \doteq 8.5, 13.5$



a) $y = x^2 - 10x + 20$

$y = x^2 - 10x + 25 - 25 + 20$

$y = (x - 5)^2 - 5$

4.2 Factoring Quadratic Equations, pages 156–164

4. Factor each of the following completely.

a) $(a+5)^2-49(b-9)^2$

$= [a+5-7(b-9)][a+5+7(b-9)]$

$= [a+5-7b+63][a+5+7b-63]$

$= \boxed{(a-7b+68)(a+7b-58)}$

b) $(x-6)^2+10(x-6)+9$ let $y = x-6$

$= y^2+10y+9$

$= (y+9)(y+1)$

$= (x-6+9)(x-6+1)$

$= \boxed{(x+3)(x-5)}$

c) $\frac{9m^2}{16}-\frac{100n^2}{81}$

$= \boxed{\left[\frac{3m}{4}-\frac{10n}{9}\right]\left[\frac{3m}{4}+\frac{10n}{9}\right]}$

5. Solve each of the following equations by factoring. Verify your answers.

a) $x^2+6x+8=0$

$(x+4)(x+2)=0$

$x+4=0 \quad x+2=0$

$\boxed{x=-4} \quad \boxed{x=-2}$

b) $3x^2-5x+2=0$

$3x^2-3x-2x+2=0$

$3x(x-1)-2(x-1)=0$

$(3x-2)(x-1)=0$

$3x-2=0 \quad x-1=0$

$3x=2 \quad \boxed{x=1}$

$\boxed{x=2/3}$

c) $4x^2+27=24x$

108 1
54 2
36 3
27 4
18 6
12 9

$4x^2-24x+27=0$

$4x^2-18x-6x+27=0$

$2x(2x-9)-3(2x-9)=0$

$(2x-3)(2x-9)=0$

$2x-3=0 \quad 2x-9=0$

$2x=3 \quad 2x=9$

$\boxed{x=3/2} \quad \boxed{x=9/2}$

d) $36x^2-81=0$

$9(4x^2-9)=0$

$9(2x+3)(2x-3)=0$

$2x+3=0 \quad 2x-3=0$

$2x=-3 \quad 2x=3$

$\boxed{x=-3/2} \quad \boxed{x=3/2}$

6. One side of an envelope is 3 inches longer than the other side. The area of the envelope is 108 in.^{2}. Determine the dimensions of the envelope. (Sketch a diagram to help you with your solution.)

$A = 108$, w, $l = w+3$

$108 = w(w+3)$

$0 = w^2 + 3w - 108$

$0 = (w+12)(w-9)$

$w = -12, 9$

width = 9″

length = 9 + 3 = 12″

4.3 Solving Quadratic Equations by Completing the Square, pages 165–171

7. Solve each of the following. State your answers as exact values.

a) $\sqrt{x^2} = \sqrt{169}$

$x = \pm 13$

b) $\sqrt{(x+7)^2} = \sqrt{121}$

$x + 7 = \pm 11$

$x = -7 \pm 11$

$x = -18, 4$

c) $\sqrt{(x-12)^2} = \sqrt{80}$

$x - 12 = \pm\sqrt{80}$

$x = 12 \pm \sqrt{80}$

or $x = 12 \pm 4\sqrt{5}$

d) $-3(x+1)^2 = -48$

$\sqrt{(x+1)^2} = \sqrt{16}$

$x + 1 = \pm 4$

$x = -1 \pm 4$

$x = -5, 3$

8. Solve each of the following by completing the square. State your answers as exact values and as approximations to the nearest tenth.

a) $x^2 + 8x = 7$

$x^2 + 8x - 7 = 0$

$x^2 + 8x + 16 - 16 - 7 = 0$

$(x+4)^2 - 23 = 0$

$\sqrt{(x+4)^2} = \sqrt{23}$

$x + 4 = \pm\sqrt{23}$

$x = -4 \pm \sqrt{23}$

$x \doteq 0.8, -8.8$

b) $2x^2 - 20x + 14 = 0$

$2(x^2 - 10x + 7) = 0$

$x^2 - 10x + 25 - 25 + 7 = 0$

$(x-5)^2 - 18 = 0$

$\sqrt{(x-5)^2} = \sqrt{18}$

$x - 5 = \pm\sqrt{18}$

$x = 5 \pm \sqrt{18}$

$x \doteq 9.2, 0.8$

9. The profit, p, earned from the sale of a particular product by a business is given by $p(d) = -0.25d^2 + 5d + 80$, where d is the number of days the product has been for sale. Solve this equation by completing the square to determine the last day on which the product will be profitable.

$0 = -0.25d^2 + 5d + 80$

$0 = -0.25(d^2 - 20d - 320)$

$0 = d^2 - 20d + 100 - 100 - 320$

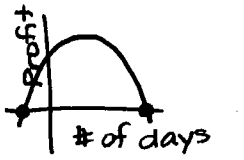
$0 = (d - 10)^2 - 420$

$\sqrt{(d - 10)^2} = \sqrt{420}$

$d - 10 = \pm\sqrt{420}$

$d = 10 \pm \sqrt{420}$

$d \doteq 30.5, -10.5$



Day 30

4.4 The Quadratic Formula, pages 172–180

10. Use the discriminant to decide the nature of the roots for each of the following.

What is the discriminant? What can it tell you?

a) $2x^2 + 5x = 8 \rightarrow 2x^2 + 5x - 8 = 0$

$b^2 - 4ac = 5^2 - 4(2)(-8)$

$= 25 + 64$

$= 89 \rightarrow$ positive, so 2 roots

b) $x^2 = x + 12 \rightarrow x^2 - x - 12 = 0$

$b^2 - 4ac = (-1)^2 - 4(1)(-12)$

$= 1 + 48$

$= 49 \rightarrow$ positive, so 2 roots

c) $16x^2 + 49 = -56x \rightarrow 16x^2 + 56x + 49 = 0$

$b^2 - 4ac = 56^2 - 4(16)(49)$

$= 3136 - 3136$

$= 0 \rightarrow$ zero, so 1 root

d) $7x^2 = 3x - 2 \rightarrow 7x^2 - 3x + 2 = 0$

$b^2 - 4ac = (-3)^2 - 4(7)(2)$

$= 9 - 56$

$= -47 \rightarrow$ negative, so no roots

11. Use the quadratic formula to solve each of the following. State your answers as exact values and as approximations to the nearest tenth.

a) $x^2 + 10 = 10x$ → $x^2 - 10x + 10$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{60}}{2}$$

$x \doteq 8.9, 1.1$

b) $5x^2 = 8 - 2x$ → $5x^2 + 2x - 8 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(5)(-8)}}{2(5)}$$

$$x = \frac{-2 \pm \sqrt{164}}{10}$$

$x \doteq 1.1, -1.5$

12. Solve each of the following using an algebraic method. Explain your choice of method.

a) $x^2 + 4x = 21$

$x^2 + 4x - 21 = 0$

$(x + 7)(x - 3) = 0$

$x = -7, 3$

30 1
15 2
10 3
6 5

b) $5x^2 - 13x - 6 = 0$

$5x^2 - 15x + 2x - 6 = 0$

$5x(x - 3) + 2(x - 3) = 0$

$(5x + 2)(x - 3) = 0$

$x = -2/5, 3$

6 1
3 2

c) $2x^2 + 9x = -3$

$2x^2 + 9x + 3 = 0$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-9 \pm \sqrt{57}}{4}$$

$x \doteq -0.4, -4.1$