

Chapter 4 Review

4.4 1. Write each power as a radical.

a) $13^{\frac{1}{2}} = \sqrt{13}$

b) $(-2)^{\frac{1}{3}} = \sqrt[3]{-2}$

c) $15^{\frac{1}{4}} = \sqrt[4]{15}$

2. Evaluate each power.

a) $25^{\frac{1}{2}} = \sqrt{25} = 5$

b) $(-1000)^{\frac{1}{3}} = \sqrt[3]{-1000} = -10$

c) $1^{\frac{1}{4}} = \sqrt[4]{1} = 1$

3. This formula is used to estimate the thickness of ice, t millimetres, needed to support a mass, m kilograms, safely: $t = 3.8 m^{\frac{1}{2}}$

To the nearest millimetre, estimate the thickness of ice needed to support each mass.

a) a person with a mass of 75 kg

$$m = 75 \rightarrow t = 3.8 \times 75^{\frac{1}{2}}$$

$$t = 3.8 \times \sqrt{75}$$

The thickness is about: 32.9 mm

b) a car with a mass of 800 kg

$$m = 800 \rightarrow t = 3.8 \times 800^{\frac{1}{2}}$$

$$t = 3.8 \times \sqrt{800}$$

The thickness is about: 107.5 mm = 10.75 cm

4. Write each radical as a power with a fractional exponent.

a) $\sqrt{5^3} = 5^{\frac{3}{2}}$

b) $(\sqrt{27})^5 = 27^{\frac{5}{2}}$

c) $\sqrt[3]{25^4} = 25^{\frac{4}{3}}$

d) $(\sqrt[4]{11})^3 = 11^{\frac{3}{4}}$

5. Evaluate each expression.

a) $(\sqrt[3]{-8})^2 = (-2)^2 = 4$

b) $\sqrt{4^3} = 2^3 = 8$

c) $(-64)^{\frac{2}{3}} = \sqrt[3]{-64}^2$
 $= (-4)^2 = 16$

d) $81^{\frac{5}{4}} = \sqrt[4]{81}^5 = 3^5$
 $= 243$

4.4 10. Evaluate each power.

a) $36^{\frac{1}{2}} = \sqrt{36} = 6$ b) $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$ c) $81^{\frac{3}{4}} = \sqrt[4]{81^3} = 3^3 = 27$

11. a) Write each radical as a power.

i) $\sqrt[5]{18^2} = 18^{\frac{2}{5}}$

ii) $(\sqrt[4]{21})^3 = 21^{\frac{3}{4}}$

b) Write each power as a radical.

i) $25^{\frac{4}{3}} = \sqrt[3]{25^4}$

ii) $75^{\frac{2}{5}} = \sqrt[5]{75^2}$

4.5 6. Write each power with a positive exponent.

a) $7^{-3} = \frac{1}{7^3}$

b) $\frac{1}{2^{-3}} = 2^3$

c) $5^{-\frac{3}{2}} = \frac{1}{5^{3/2}} = \frac{1}{\sqrt{5^3}}$

d) $\left(\frac{4}{9}\right)^{-5} = \left(\frac{9}{4}\right)^5 = \frac{9^5}{4^5}$

7. Evaluate each power.

a) $36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}}$

$= \frac{1}{\sqrt{36}}$

$= \frac{1}{6}$

b) $8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}}$

$= \frac{1}{\sqrt[3]{8}}$

$= \frac{1}{2}$

c) $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}}$

$= \frac{1}{\sqrt[3]{27^2}}$

$= \frac{1}{3^2}$

$= \frac{1}{9}$

4.5 12. Write each power with a positive exponent.

a) $8^{-5} = \frac{1}{8^5}$

b) $6^{-\frac{2}{3}} = \frac{1}{6^{\frac{2}{3}}}$

c) $\frac{1}{3^{-10}} = 3^{10}$

d) $\left(\frac{9}{2}\right)^{-3} = \left(\frac{2}{9}\right)^3$
 $= \frac{2^3}{9^3}$

13. Evaluate each power.

a) $3^{-3} = \frac{1}{3^3}$
 $= \frac{1}{27}$

b) $\left(\frac{3}{10}\right)^{-3} = \left(\frac{10}{3}\right)^3$
 $= \frac{10^3}{3^3}$
 $= \frac{1000}{27}$

c) $4^{\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}}$
 $= \frac{1}{\sqrt{4}^3}$
 $= \frac{1}{2^3}$
 $= \frac{1}{8}$

4.6 14. Evaluate.

a) $(3^{-2})^{-2} = 3^4$
 $= 81$

b) $\left(4^{\frac{1}{2}} \cdot 4^{\frac{3}{2}}\right)^{-1} = (4^2)^{-1}$
 $= 4^{-2}$
 $= \frac{1}{4^2} = \frac{1}{16}$

$\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$

15. Simplify. Write an expression with a positive exponent where necessary.

a) $x^{\frac{2}{3}} \cdot x^{-2} =$
 $= x^{-\frac{4}{3}}$
 $= \frac{1}{x^{\frac{4}{3}}}$

$\frac{2}{3} + \frac{-2}{1} = \frac{2}{3} + \frac{-6}{3}$
 $= \frac{-4}{3}$

b) $\frac{3 \cdot 18x^{\frac{1}{2}}}{4 \cdot 24x^{\frac{3}{2}}} = \frac{3x^2}{4}$

$\frac{1}{2} - \frac{3}{2} = \frac{1}{2} + \frac{3}{2}$
 $= \frac{4}{2} = 2$

1. Write as a power with a positive exponent.

a) $5^7 \cdot 5^4 \cdot 5^{-7} = 5^4$

b) $\frac{11^2}{11^{-3}} = 11^5$

c) $(3^2 \cdot 3^2)^{-2} = (3^4)^{-2}$
 $= 3^{-8}$
 $= \frac{1}{3^8}$

d) $\left(\frac{8^2}{8^3}\right)^{-4} = (8^{-1})^{-4}$
 $= 8^4$

2. Evaluate.

$$\begin{aligned} \text{a) } (7^{-2})^{-1} &= 7^2 \\ &= 49 \end{aligned}$$

$$\begin{aligned} \text{b) } 5^3 \cdot (5^{-2})^2 &= 5^3 \cdot 5^{-4} \\ &= 5^{-1} \\ &= \frac{1}{5} \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{4^{-3} \cdot 4^{-1}}{4^{-2}} \right)^2 &= \left(\frac{4^{-4}}{4^{-2}} \right)^2 \\ &= (4^{-2})^2 \\ &= 4^{-4} \\ &= \frac{1}{4^4} \\ &= \frac{1}{256} \end{aligned}$$

Each answer should be an integer or a fraction.

3. Write as a power with a positive exponent.

$$\begin{aligned} \text{a) } 5^{\frac{3}{4}} \cdot 5^{-\frac{1}{4}} &= 5^{\frac{3}{4} + (-\frac{1}{4})} = 5^{\frac{2}{4}} = 5^{\frac{1}{2}} \\ \text{b) } (7^{-0.5} \cdot 7^{2.5})^{-2} &= (7^2)^{-2} \\ &= 7^{-4} \\ &= \frac{1}{7^4} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{3^{-\frac{2}{3}}}{3^{\frac{4}{3}}} &= 3^{-2} \\ &= \frac{1}{3^2} \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{2^{-1.75}}{2^{-0.25}} \right)^3 &= (2^{-1.5})^3 \\ &= 2^{-4.5} \\ &= \frac{1}{2^{9/2}} \end{aligned}$$

$$-4.5 = -\frac{9}{2}$$